Section E.2

Activity E.2.1 *(~10 min)* Free browser-based technologies for mathematical computation are available online.

- Go to `http://www.cocalc.com` and create an account.
- Create a project titled "Linear Algebra Team X" with your appropriate team number. Add all team members as collaborators.
- Open the project and click on "New"
- Give it an appropriate name such as "Class E.2 workbook". Make a new Jupyter notebook.
- Click on "Kernel" and make sure "Octave" is selected.
- Type `A=[1 3 4 ; 2 5 7]` and press `Shift+Enter` to store the matrix $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 7 \end{bmatrix}$ in the variable A.
- Type `rref(A)` and press `Shift+Enter` to compute the reduced row echelon form of A.

Remark E.2.2 If you need to find the reduced row echelon form of a matrix during class, you are encouraged to use CoCalc's Octave interpreter.

You can change a cell from "Code" to "Markdown" or "Raw" to put comments around your calculations such as Activity numbers.

Activity E.2.3 *(~10 min)* Consider the system of equations.

$$\begin{aligned} 3x_1 - 2x_2 + 13x_3 &= 6 \\ 2x_1 - 2x_2 + 10x_3 &= 2 \\ -x_1 + 3x_2 - 6x_3 &= 11 \end{aligned}$$

Convert this to an augmented matrix and use CoCalc to compute its reduced row echelon form. Write these on your whiteboard, and use them to write a simpler yet equivalent linear system of equations. Then find its solution set.

Activity E.2.4 *(~10 min)* Consider our system of equations from above.

$$\begin{aligned} 3x_1 - 2x_2 + 13x_3 &= 6 \\ 2x_1 - 2x_2 + 10x_3 &= 2 \\ -x_1 - 3x_3 &= 1 \end{aligned}$$

Convert this to an augmented matrix and use CoCalc to compute its reduced row echelon form. Write these on your whiteboard, and use them to write a simpler yet equivalent linear system of equations. Then find its solution set.

Activity E.2.5 *(~10 min)* Consider the following linear system.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ 2x_1 + 4x_2 + 8x_3 &= 0 \end{aligned}$$

Part 1: Find its corresponding augmented matrix A and use CoCalc to find RREF(A).
Part 2: How many solutions does the corresponding linear system have?

Activity E.2.6 *(~10 min)* Consider the simple linear system equivalent to the system from the previous problem:

$$\begin{aligned} x_1 + 2x_2 \quad &= 4 \\ x_3 &= -1 \end{aligned}$$

Part 1: Let $x_1 = a$ and write the solution set in the form $\left\{ \begin{bmatrix} a \\ ? \\ ? \end{bmatrix} \middle| \, a \in \mathbb{R} \right\}$.

Part 2: Let $x_2 = b$ and write the solution set in the form $\left\{ \begin{bmatrix} ? \\ b \\ ? \end{bmatrix} \middle| \, b \in \mathbb{R} \right\}$.

Part 3: Which of these was easier? What features of the RREF matrix $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right]$ caused this?

Definition E.2.7 Recall that the pivots of a matrix in RREF form are the leading 1s in each non-zero row.

The pivot columns in an augmented matrix correspond to the **bound variables** in the system of equations. The remaining variables are called **free variables**.

To efficiently solve a system in RREF form, we may assign letters to free variables and solve for the bound variables.

Activity E.2.8 *(~10 min)* Find the solution set for the system

$$\begin{aligned} 2x_1 - 2x_2 - 6x_3 + x_4 - x_5 &= 3 \\ -x_1 + x_2 + 3x_3 - x_4 + 2x_5 &= -3 \\ x_1 - 2x_2 - x_3 + x_4 + x_5 &= 2 \end{aligned}$$

by assigning letters to the free variables and solving for the bound variables in the simplified system given by row-reducing its augmented matrix.

Observation E.2.9 The solution set to the system

$$\begin{alignedat}{6} 2x_1 &- 2x_2 &- 6x_3 &+ x_4 &- x_5 &= 3 \\ -x_1 &+ x_2 &+ 3x_3 &- x_4 &+ 2x_5 &= -3 \\ x_1 &- 2x_2 &- x_3 &+ x_4 &+ x_5 &= 2 \end{alignedat}$$

may be written as

$$\left\{ \begin{bmatrix} 1+5a+2b \\ 1+2a+3b \\ a \\ 3+3b \\ b \end{bmatrix} \middle| \, a, b \in \mathbb{R} \right\}.$$

Remark E.2.10 Don't forget to correctly express the solution set of a linear system, using set-builder notation for consistent systems with infintely many solutions.

- **Consistent with one solution**: e.g. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$
- **Consistent with infinitely-many solutions**: e.g. $\left\{ \begin{bmatrix} 1 \\ 2-3a \\ a \end{bmatrix} \middle| \, a \in \mathbb{R} \right\}$
- **Inconsistent**: $\emptyset$