1.1

(1) Use $\ln x = \int_1^x \frac{1}{t}\,dt$ to prove $\ln(x^p) = p\ln(x)$ for $x>0$ and $p \in \mathbb{Q}$.

$\frac{d}{dx}[\ln(x^p)] = \frac{1}{x^p}(px^{p-1}) = p/x$

$\frac{d}{dx}[p\ln(x)] = p\,\frac{1}{x} = p/x$

Thus $\ln(x^p) = p\ln(x) + C$.

For $x=1$,

$\ln(1^p) = p\ln(1) + C$

$\ln(1) = p(0) + C$

$0 = C$.

Therefore $\ln(x^p) = p\ln(x)$. □

(2) Find $\int 6/x^3 + 2/x - 3x\,dx$.

$= \int 6x^{-3} + 2\frac{1}{x} - 3x\,dx$

$= 6/-2x^{-2} + 2\ln|x| - 3/2x^2 + C$

$= -3/x^2 + 2\ln|x| - 3/2x^2 + C$

(3) Find $\int \frac{6x^4 - x^2 + 4}{2x^3}\,dx$.

$$= \int \frac{6x^4}{2x^3} - \frac{x^2}{2x^3} + \frac{4}{2x^3}\,dx$$

$$= \int 3x - \frac{1}{2}\frac{1}{x} + 2x^{-3}\,dx$$

$$= 3\left(\tfrac{1}{2}x^2\right) - \tfrac{1}{2}\ln|x| + 2\left(-\tfrac{1}{2}x^{-2}\right) + C$$

$$= \frac{3}{2}x^2 - \frac{1}{2}\ln|x| - \frac{1}{x^2} + C$$

(4) Describe infinitely many functions $f(x)$ such that $f'(x) = f(x)$.

Let k be any constant real number, and $f(x) = ke^x$. Then

$$f'(x) = \frac{d}{dx}[ke^x]$$
$$= k\frac{d}{dx}[e^x]$$
$$= ke^x$$
$$= f(x). \quad \square$$

(5) Find $\frac{d}{dx}\left[\frac{1}{x} + 3e^x\right]$.

$$= \frac{d}{dx}\left[x^{-1} + 3e^x\right]$$
$$= -x^{-2} + 3e^x$$
$$= -\frac{1}{x^2} + 3e^x$$

---

(6) Prove the derivative formulas:

$$\frac{d}{dx}\left[\log_b x\right] = \frac{1}{x \ln b} \quad \text{and} \quad \frac{d}{dx}\left[a^x\right] = a^x \ln a$$

$$\frac{d}{dx}\left[\log_b x\right] = \frac{d}{dx}\left[\frac{\ln x}{\ln b}\right] = \frac{1}{\ln b}\frac{d}{dx}\left[\ln x\right] = \frac{1}{\ln b} \cdot \frac{1}{x}$$
$$= \frac{1}{x \ln b}$$

$$\frac{d}{dx}\left[a^x\right] = \frac{d}{dx}\left[e^{(x \ln a)}\right] = e^{(x \ln a)}\left(1 \ln a\right)$$
$$= a^x \ln a. \quad \square$$

(7) (Quiz) Find $\int 3x^4 + 3e^x - 4/x \, dx$.

$$= 3\left(\tfrac{1}{5}x^5\right) + 3e^x - 4\ln|x| + C$$
$$= \tfrac{3}{5}x^5 + 3e^x - 4\ln|x| + C.$$

(8) (Quiz) Differentiate $f(x) = \ln(x^2) + e^{x^3}$.

$$f'(x) = \frac{1}{x^2}(2x) + e^{x^3}(3x^2)$$
$$= 2/x + 3x^2 e^{x^3}.$$