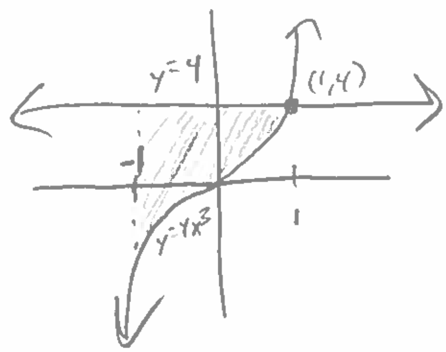
1. Find the area between the curves $y=4$ and $y=4x^3$ from -1 to 1.

Sketch:



$$A=\int_{-1}^{1}\left[(4)-(4x^3)\right]dx$$

$$=\left[4x-x^4\right]_{-1}^{1}$$

$$=(4-1)-(-4-1)$$

$$=\boxed{8}$$

② Find the area bounded by the curves $y = x^2 - 2x$ and $y = x$.

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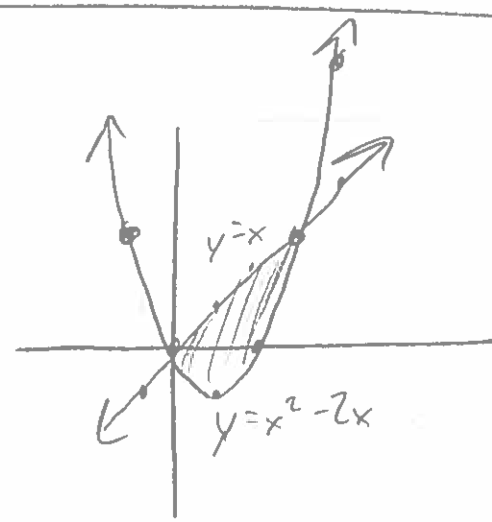
x	$y = x^2 - 2x$	$y = x$
-1	3	-1
0	0	0
1	-1	1
2	0	2
3	3	3
4	8	4



OR

$$y = x^2 - 2x = x$$
$$x^2 - 3x = 0$$
$$x(x-3) = 0$$

Cross @ $x=0$ & $x=3$

$$A = \int_0^3 \left[(x) - (x^2 - 2x)\right] dx$$

$$= \int_0^3 3x - x^2 \, dx$$

$$= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$$

$$= \left(\frac{27}{2} - 9\right) - \cancel{(0 - 0)}$$

$$= \boxed{9/2}$$

3) Find the area bounded by the curves $y = \pm\sqrt{4-x}$ and $x = 3$.

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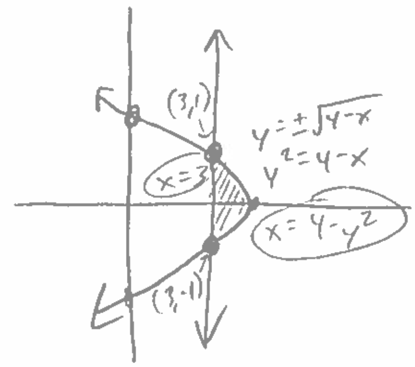
x	$y = \pm\sqrt{4-x}$	$y = 3$
0	± 2	3
3	± 1	3
4	0	3



$$A = \int_{-1}^{1} \left[(4-y^2) - (3) \right] dy$$

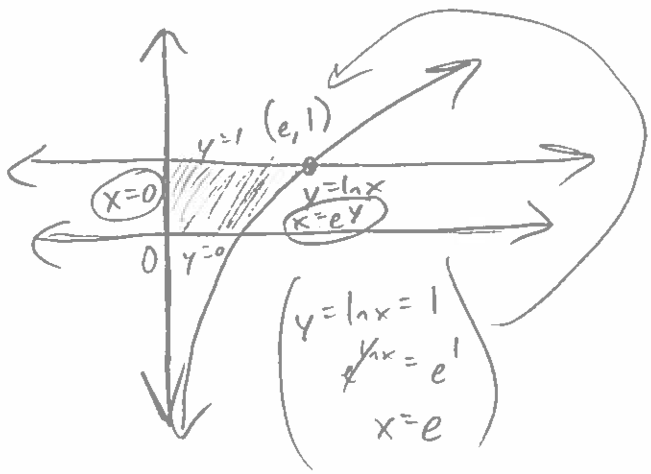
$$= \int_{-1}^{1} (1-y^2)\, dy$$

$$= \left[y - \tfrac{1}{3}y^3 \right]_{-1}^{1}$$

$$= \left(1 - \tfrac{1}{3}\right) - \left(-1 + \tfrac{1}{3}\right)$$

$$= 2 - \tfrac{2}{3} = \boxed{4/3}$$

④ Find the area bounded by the curves $y=0$, $x=0$, $y=1$, and $y=\ln x$.



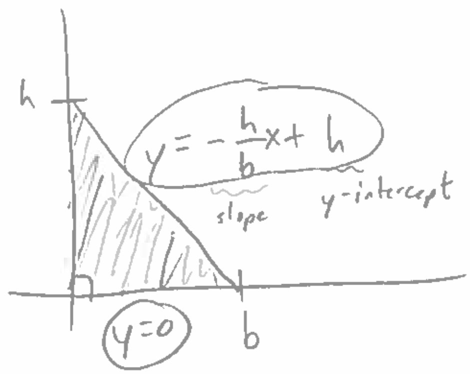
$$A = \int_0^1 [(e^y)-(0)]\,dy$$

$$= \left[e^y\right]_0^1$$

$$= e^1 - e^0$$

$$= \boxed{e-1}$$

(5) Prove that the area of the triangle with vertices $(0,0)$, $(b,0)$, $(0,h)$ is $\frac{1}{2}bh$.



$$A = \int_0^b \left[\left(-\frac{h}{b}x + h\right) - (0)\right] dx$$

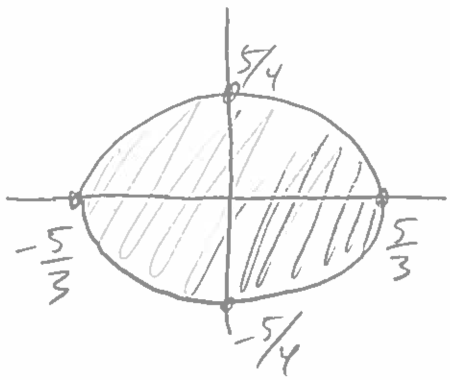
$$= \left[-\frac{h}{b}\frac{1}{2}x^2 + hx\right]_0^b$$

$$= \left(-\frac{h}{\cancel{b}}\frac{1}{2}b^{\cancel{2}} + hb\right) - \cancel{(0+0)}$$

$$= hb - \frac{1}{2}hb$$

$$= \boxed{\frac{1}{2}bh} \qquad \square$$

⑥ Find the area of the ellipse bounded by $9x^2+16y^2=25$.



$$y=\pm\frac{1}{4}\sqrt{25-9x^2}$$

$$A=\int_{-5/3}^{5/3}\left[\left(\frac{1}{4}\sqrt{25-9x^2}\right)-\left(-\frac{1}{4}\sqrt{25-9x^2}\right)\right]dx$$

$$=\frac{1}{2}\int_{-5/3}^{5/3}\sqrt{25-9x^2}\,dx$$

Let $25-9x^2=25-25\sin^2\theta=25\cos^2\theta$

$x=\frac{5}{3}\sin\theta$

$dx=\frac{5}{3}\cos\theta\,d\theta$

$x=5/3 \rightarrow 5/3=5/3\sin\theta$
$1=\sin\theta$
$\theta=\pi/2$

$x=-5/3 \rightarrow -5/3=5/3\sin\theta$
$-1=\sin\theta$
$\theta=-\pi/2$

$$=\frac{1}{2}\int_{-\pi/2}^{\pi/2}\sqrt{25\cos^2\theta}\,\frac{5}{3}\cos\theta\,d\theta$$

$$=\frac{25}{6}\int_{-\pi/2}^{\pi/2}\cos^2\theta\,d\theta$$

(cont.)

$$= \frac{25}{6} \int_{-\pi/2}^{\pi/2} \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta \right) d\theta$$

$$= \frac{25}{6} \left[\frac{1}{2}\theta + \frac{1}{4} \sin 2\theta \right]_{-\pi/2}^{\pi/2}$$

$$= \frac{25}{6} \left[\left(\frac{1}{2} \frac{\pi}{2} + \cancel{\overset{0}{\frac{1}{4} \sin \pi}} \right) - \left(\frac{1}{2}\left(-\frac{\pi}{2}\right) + \cancel{\overset{0}{\frac{1}{4} \sin(-\pi)}} \right) \right]$$

$$= \frac{25}{6} \left[\frac{\pi}{2} \right] = \boxed{\frac{25}{12}\pi}$$