1. Find the volume of a solid located between $x=-1$ and $x=2$ where $A(x)=x^2+1$.

$$V=\int_a^b A(x)\,dx$$

$$=\int_{-1}^{2}(x^2+1)\,dx$$

$$=\left[\frac{1}{3}x^3+x\right]_{-1}^{2}$$

$$=\left(\frac{8}{3}+2\right)-\left(-\frac{1}{3}-1\right)$$

$$=\frac{14}{3}+\frac{4}{3}$$

$$=\frac{18}{3}=\boxed{6}$$

(2) Find the volume of a solid located between $x=0$ and $x=1$ whose cross-sections are parallelograms with base length $b(x)=x+1$ and height $h(x)=x^2+1$ for all $0 \le x \le 1$.

$$V = \int_a^b A(x)\,dx$$

(Area of parallelogram = base × height)

$$= \int_0^1 b(x)\,h(x)\,dx$$

$$= \int_0^1 (x+1)(x^2+1)\,dx$$

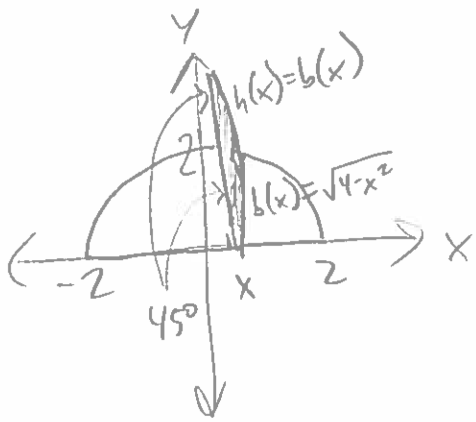
$$= \int_0^1 \left(x^3+x^2+x+1\right)dx$$

$$= \left[\frac{1}{4}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2+x\right]_0^1$$

$$= \frac{1}{4}+\frac{1}{3}+\frac{1}{2}+1$$

$$= \frac{3+4+6+12}{12} = \boxed{\frac{25}{12}}$$

(3) Find the volume of a wedge cut from a circular cylindar with radius 2, sliced out at a 45° angle from the diameter of its base.



$$V = \int_a^b A(x)\,dx$$

$$= \int_{-2}^{2} \frac{1}{2} b(x)\, h(x)\, dx$$

$$= \int_{-2}^{2} \frac{1}{2}\sqrt{4-x^2}\sqrt{4-x^2}\, dx$$

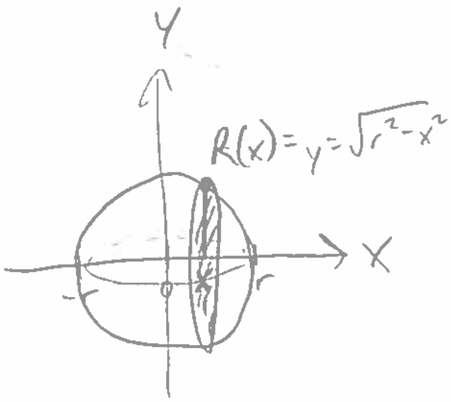
$$= \int_{-2}^{2} \left(2 - \frac{1}{2}x^2\right) dx$$

$$= \left[2x - \frac{1}{6}x^3\right]_{-2}^{2}$$

$$= \left(4 - \frac{8}{6}\right) - \left(-4 + \frac{8}{6}\right)$$

$$= 8 - \frac{8}{3} = \boxed{16/3}$$

④ Prove that the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.



$$V = \int_{-r}^{r} \pi (R(x))^2 \, dx$$

$$= \int_{-r}^{r} (\pi r^2 - \pi x^2) \, dx$$

$$= \left[\pi r^2 x - \frac{1}{3}\pi x^3\right]_{-r}^{r}$$

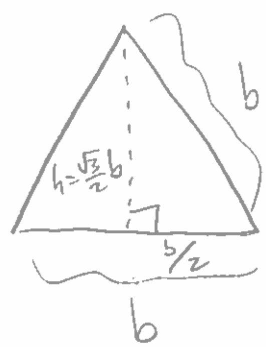
$$= \left(\pi r^3 - \frac{1}{3}\pi r^3\right) - \left(-\pi r^3 + \frac{1}{3}\pi r^3\right)$$

$$= \pi r^3 \left[(1 - 1/3) - (-1 + 1/3)\right]$$

$$= \boxed{4/3\, \pi r^3} \quad \square$$

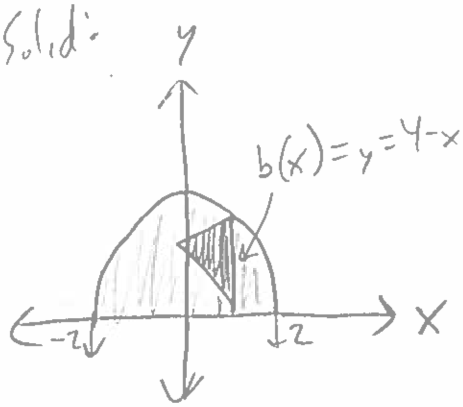
⑤ Find the volume of the solid whose base is the region $0 \le y \le 4-x^2$ and whose cross-sections are equilateral triangles perpindicular to the x-axis.

Equilateral triangle:



$$A(x)=\frac{1}{2}b(x)h(x)$$
$$=\frac{\sqrt{3}}{4}b(x)^2$$

3D Solid:



$$A(x)=\frac{\sqrt{3}}{4}(4-x^2)^2$$
$$=\frac{\sqrt{3}}{4}(16-8x+x^2)$$

$$V=\frac{\sqrt{3}}{4}\int_{-2}^{2}(16-8x+x^2)\,dx$$

$$=\frac{\sqrt{3}}{4}\left[16x-4x^2+\frac{1}{3}x^3\right]_{-2}^{2}$$

$$=\frac{\sqrt{3}}{4}\left[\left(32-\cancel{16}+\frac{8}{3}\right)-\left(-32-\cancel{16}-\frac{8}{3}\right)\right]$$

$$=\frac{\sqrt{3}}{4}\left[64+\frac{16}{3}\right]$$

$$=\sqrt{3}\left[16+\frac{4}{3}\right]=\boxed{\frac{52\sqrt{3}}{3}}$$