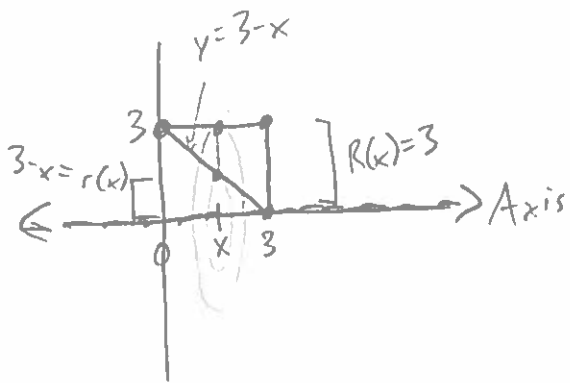
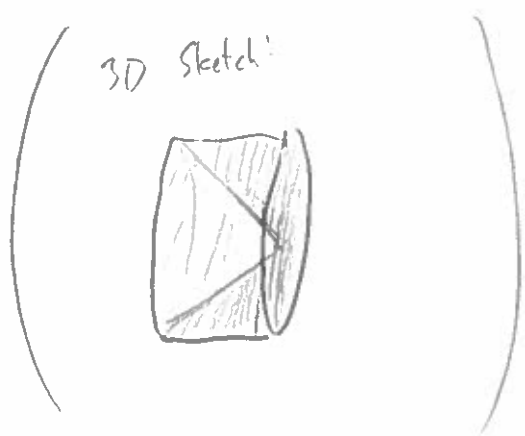


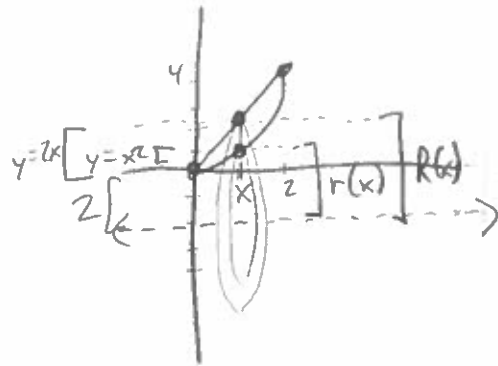
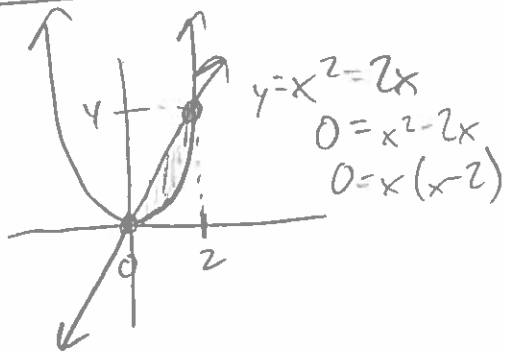
① Find the volume of the solid of revolution obtained by rotating the triangle with vertices  $(3,0)$ ,  $(3,3)$ ,  $(0,3)$  around the  $x$ -axis.



$$\begin{aligned}
 V &= \pi \int_0^3 \left( [3]^2 - [3-x]^2 \right) dx \\
 &= \pi \int_0^3 \left( 9 - (9 - 6x + x^2) \right) dx \\
 &= \pi \int_0^3 (6x - x^2) dx \\
 &= \pi \left[ 3x^2 - \frac{1}{3}x^3 \right]_0^3 \\
 &= \pi \left[ (27 - 9) - (0) \right] \\
 &= \boxed{18\pi}
 \end{aligned}$$



② Find the volume of the solid of revolution obtained by rotating the region bounded by  $y=x^2$ ,  $y=2x$  around the axis  $y=-2$ .



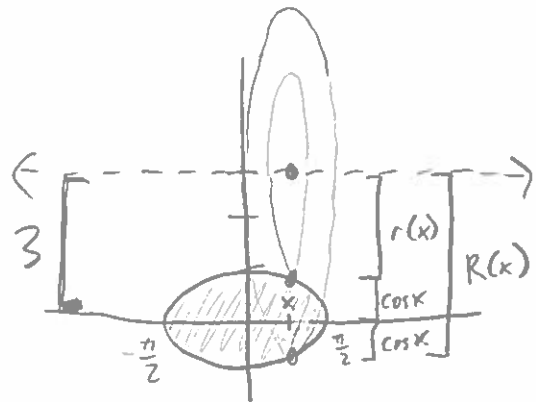
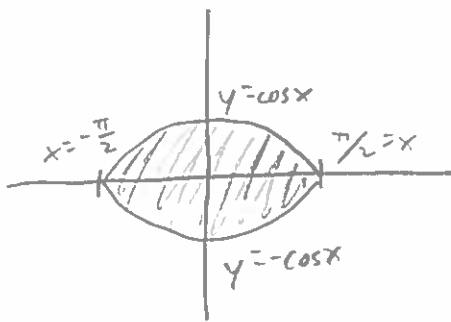
$$R(x) = 2 + 2x$$

$$r(x) = 2 + x^2$$

$$\begin{aligned}
 V &= \pi \int_0^2 \left( [2+2x]^2 - [2+x^2]^2 \right) dx \\
 &= \pi \int_0^2 \left( (4+8x+4x^2) - (4+4x^2+x^4) \right) dx \\
 &= \pi \int_0^2 (8x - x^4) dx \\
 &= \pi \left[ 4x^2 - \frac{1}{5}x^5 \right]_0^2 \\
 &= \pi \left[ 16 - \frac{32}{5} \right] = \boxed{\frac{48\pi}{5}}
 \end{aligned}$$

3) Consider the region in the  $xy$  plane satisfying  $|x| \leq \frac{\pi}{2}$  and  $|y| \leq \cos x$ . Find the volume of the solid of revolution obtained by rotating this region around the axis  $y=3$ .

$$-\cos x \leq y \leq \cos x$$



$$3 = r(x) + \cos(x)$$

$$r(x) = 3 - \cos(x)$$

$$R(x) = 3 + \cos(x)$$

$$V = \pi \int_{-\pi/2}^{\pi/2} \left( [3 + \cos x]^2 - [3 - \cos x]^2 \right) dx$$

$$= \pi \int_{-\pi/2}^{\pi/2} \left( [9 + 6\cos x + \cancel{\cos^2 x}] - [9 - 6\cos x + \cancel{\cos^2 x}] \right) dx$$

$$= 12\pi \int_{-\pi/2}^{\pi/2} \cos x dx = 12\pi \left[ \sin x \right]_{-\pi/2}^{\pi/2} = 12\pi \left[ \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) \right]$$

$$= 12\pi [1 - (-1)] = \boxed{24\pi}$$