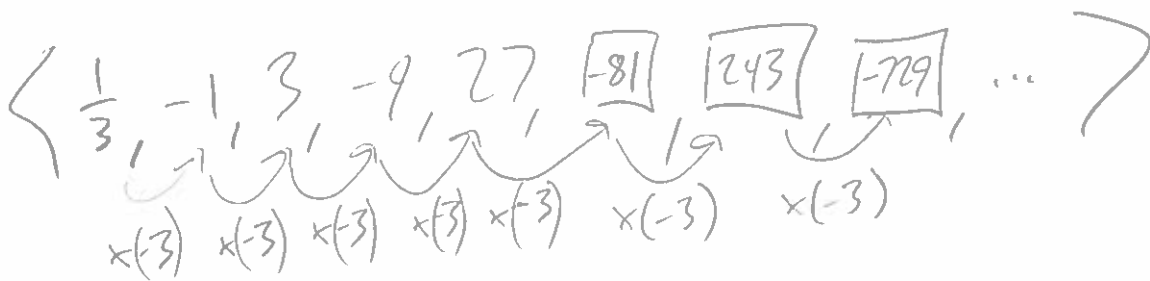
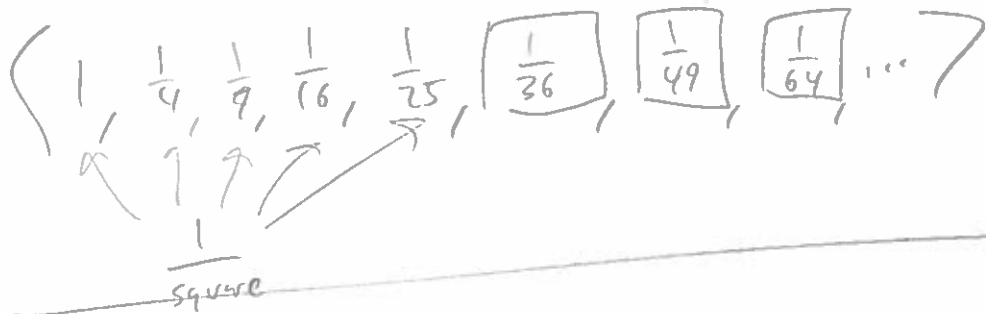
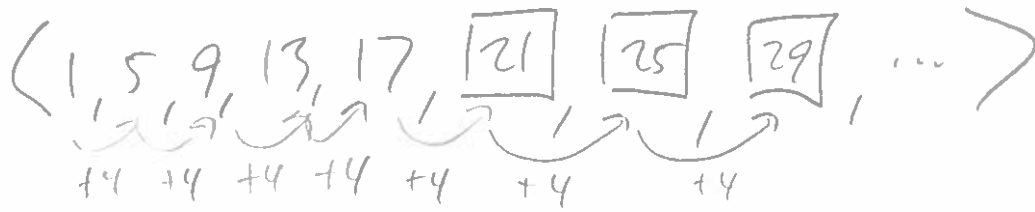
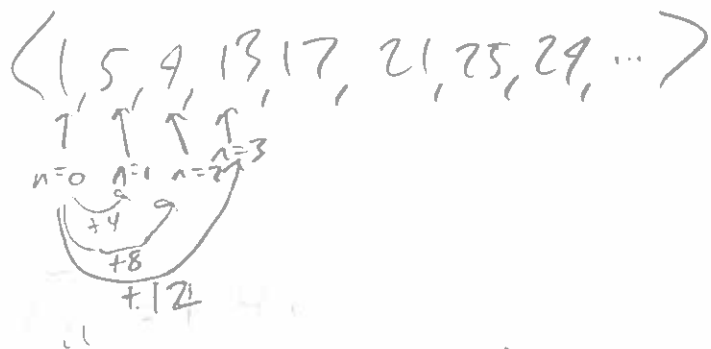


① Guess the next three terms of the sequences...



② Create an explicit formula for each of these sequences...

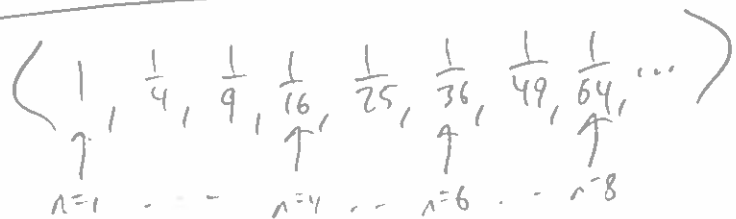


$$a_n = 1 + 4n$$

(starts with  $n=0$ )

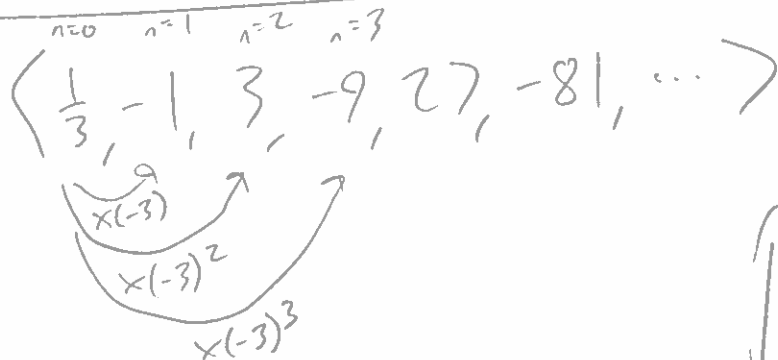
(Recursive formula:

$$a_0 = 1 \quad a_{n+1} = a_n + 4$$



$$b_n = \frac{1}{n^2}$$

(squares on bottom)



$$c_n = \frac{1}{3} (-3)^n$$

(Recursive:

$$c_0 = \frac{1}{3} \quad c_{n+1} = -3c_n$$

(3) Write the first five terms of the sequences  $\langle a_n \rangle_{n=0}^{\infty}$ ,  $\langle b_n \rangle_{n=0}^{\infty}$ ,  $\langle c_n \rangle_{n=0}^{\infty}$  defined by...

---

$$a_n = 3n + 2$$

$$\begin{aligned} & \langle 3(0)+2, 3(1)+2, 3(2)+2, 3(3)+2, 3(4)+2, \dots \rangle \\ & = \langle 2, 5, 8, 11, 14, \dots \rangle \end{aligned}$$

---

$$b_n = 2(-1/3)^n$$

$$\begin{aligned} & = \langle 2(-1/3)^0, 2(-1/3)^1, 2(-1/3)^2, 2(-1/3)^3, 2(-1/3)^4, \dots \rangle \\ & = \langle 2, -2/3, 2/9, -2/27, 2/81, \dots \rangle \end{aligned}$$

---

$$c_n = \frac{n}{1+n^2}$$

$$\begin{aligned} & = \langle \frac{0}{1+0^2}, \frac{1}{1+1^2}, \frac{2}{1+2^2}, \frac{3}{1+3^2}, \frac{4}{1+4^2}, \dots \rangle \\ & = \langle 0, \frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \dots \rangle \end{aligned}$$

(4) Write the first six terms of  $\{q_n\}_{n=0}^{\infty}$  defined by  $q_0=0$  and  $q_{n+1}=q_n+2n+1$ .

---

$$q_0 = 0$$

$$q_1 = q_{0+1} = q_0 + 2(0) + 1 = 0 + 0 + 1 = 1$$

$$q_2 = q_{1+1} = q_1 + 2(1) + 1 = 1 + 2 + 1 = 4$$

$$q_3 = q_{2+1} = q_2 + 2(2) + 1 = 4 + 4 + 1 = 9$$

$$q_4 = q_{3+1} = q_3 + 2(3) + 1 = 9 + 6 + 1 = 16$$

$$q_5 = q_{4+1} = q_4 + 2(4) + 1 = 16 + 8 + 1 = 25$$

$(0, 1, 4, 9, 16, 25, \dots)$

---

(5) Prove that  $q_n = n^2$  is an explicit formula for the previous sequence.

---

Prove  $q_0 = 0$ .

$$q_0 = 0^2 = 0. \checkmark$$

Prove  $q_{n+1} = q_n + 2n + 1$ :

$$q_{n+1} = (n+1)^2$$

$$= n^2 + 2n + 1$$

$$= q_n + 2n + 1 \checkmark$$

⑥ Write the first six terms of  $(b_n)_{n=1}^{\infty}$  defined by  $b_1 = 4$  and  $b_{n+1} = b_n/2$ .

---

$$b_1 = 4$$

$$b_2 = b_{1+1} = b_1/2 = 4/2 = 2$$

$$b_3 = b_{2+1} = b_2/2 = 2/2 = 1$$

$$b_4 = \text{~~~~~} = 1/2$$

$$b_5 = \text{~~~~~} = 1/4$$

$$b_6 = \text{~~~~~} = 1/8$$

$$\boxed{\langle 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots \rangle}$$

⑦ Prove that  $b_n = \frac{8}{2^n}$  is an explicit formula for the previous sequence.

---

$$\text{Prove } b_1 = 4:$$

$$b_1 = \frac{8}{2^1} = \frac{8}{2} = 4 \checkmark$$

$$\text{Prove } b_{n+1} = b_n/2$$

$$b_{n+1} = \frac{8}{2^{n+1}}$$

$$= \frac{1}{2} \frac{8}{2^n}$$

$$= \frac{1}{2} b_n$$

$$= b_n/2 \checkmark$$