① Guess the next three terms of the sequences...

$$\left\langle 1, 5, 9, 13, 17, \boxed{21}, \boxed{25}, \boxed{29}, \ldots \right\rangle$$

+4 +4 +4 +4 +4 +4 +4

$$\left\langle 1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \boxed{\frac{1}{36}}, \boxed{\frac{1}{49}}, \boxed{\frac{1}{64}}, \ldots \right\rangle$$

$\frac{1}{\text{square}}$

$$\left\langle \frac{1}{3}, -1, 3, -9, 27, \boxed{-81}, \boxed{243}, \boxed{-729}, \ldots \right\rangle$$

×(-3) ×(-3) ×(-3) ×(3) ×(-3) ×(-3) ×(-3)

② Create an explicit formula for each of these sequences...

$\langle 1, 5, 9, 13, 17, 21, 25, 29, \dots \rangle$

$n=0 \quad n=1 \quad n=2 \quad n=3$

$+4 \quad +8 \quad +12$

$$\boxed{a_n = 1 + 4n}$$

(starts with $n=0$)

(Recursive formula:

$a_0 = 1 \qquad a_{n+1} = a_n + 4$)

$\langle 1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \frac{1}{49}, \frac{1}{64}, \dots \rangle$

$n=1 \; \dots \; n=4 \; \dots \; n=6 \; \dots \; n=8$

$$\boxed{b_n = \frac{1}{n^2}}$$

(squares on bottom)

$n=0 \quad n=1 \quad n=2 \quad n=3$

$\langle \frac{1}{3}, -1, 3, -9, 27, -81, \dots \rangle$

$\times(-3) \quad \times(-3)^2 \quad \times(-3)^3$

$$\boxed{c_n = \frac{1}{3}(-3)^n}$$

(Recursive:

$c_0 = \frac{1}{3} \qquad c_{n+1} = -3c_n$)

(3) Write the first five terms of the sequences $\langle a_n \rangle_{n=0}^{\infty}$, $\langle b_n \rangle_{n=0}^{\infty}$, $\langle c_n \rangle_{n=0}^{\infty}$ defined by...

$\boxed{a_n = 3n+2}$

$\langle 3(0)+2,\ 3(1)+2,\ 3(2)+2,\ 3(3)+2,\ 3(4)+2, \dots \rangle$

$= \langle 2,\ 5,\ 8,\ 11,\ 14, \dots \rangle$

$\boxed{b_n = 2(-1/3)^n}$

$= \langle 2(-1/3)^0,\ 2(-1/3)^1,\ 2(-1/3)^2,\ 2(-1/3)^3,\ 2(-1/3)^4, \dots \rangle$

$= \langle 2,\ -2/3,\ 2/9,\ -2/27,\ 2/81, \dots \rangle$

$\boxed{c_n = \frac{n}{1+n^2}}$

$= \langle \frac{0}{1+0^2},\ \frac{1}{1+1^2},\ \frac{2}{1+2^2},\ \frac{3}{1+3^2},\ \frac{4}{1+4^2}, \dots \rangle$

$= \langle 0,\ \frac{1}{2},\ \frac{2}{5},\ \frac{3}{10},\ \frac{4}{17}, \dots \rangle$

(4) Write the first six terms of $\langle q_n \rangle_{n=0}^{\infty}$ defined by $q_0 = 0$ and $q_{n+1} = q_n + 2n + 1$.

$q_0 = 0$

$q_1 = q_{0+1} = q_0 + 2(0) + 1 = 0 + 0 + 1 = 1$

$q_2 = q_{1+1} = q_1 + 2(1) + 1 = 1 + 2 + 1 = 4$

$q_3 = q_{2+1} = q_2 + 2(2) + 1 = 4 + 4 + 1 = 9$

$q_4 = q_{3+1} = q_3 + 2(3) + 1 = 9 + 6 + 1 = 16$

$q_5 = q_{4+1} = q_4 + 2(4) + 1 = 16 + 8 + 1 = 25$

$\langle 0, 1, 4, 9, 16, 25, \ldots \rangle$

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(5) Prove that $q_n = n^2$ is an explicit formula for the previous sequence.

Prove $q_0 = 0$.

$q_0 = 0^2 = 0$. ✓

Prove $q_{n+1} = q_n + 2n + 1$:

$$\begin{aligned} q_{n+1} &= (n+1)^2 \\ &= n^2 + 2n + 1 \\ &= q_n + 2n + 1 \checkmark \end{aligned}$$

(6) Write the first six terms of $\langle b_n \rangle_{n=1}^{\infty}$ defined by $b_1 = 4$ and $b_{n+1} = b_n/2$.

$b_1 = 4$

$b_2 = b_{1+1} = b_1/2 = 4/2 = 2$

$b_3 = b_{2+1} = b_2/2 = 2/2 = 1$

$b_4 = \quad = 1/2$

$b_5 = \quad = 1/4$

$b_6 = \quad = 1/8$

$$\boxed{\langle 4, 2, 1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \dots \rangle}$$

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(7) Prove that $b_n = \frac{8}{2^n}$ is an explicit formula for the previous sequence.

$\boxed{\text{Prove } b_1 = 4:}$

$b_1 = \frac{8}{2^1} = \frac{8}{2} = 4 \checkmark$

$\boxed{\text{Prove } b_{n+1} = b_n/2}$

$$b_{n+1} = \frac{8}{2^{n+1}}$$

$$= \frac{1}{2}\frac{8}{2^n}$$

$$= \frac{1}{2} b_n$$

$$= b_n/2 \checkmark$$