① Write out the first four terms of the partial sum sequence for $\langle 1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \dots \rangle$.

$= \langle 1, 1-\frac{1}{3}, 1-\frac{1}{3}+\frac{1}{9}, 1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}, \dots \rangle$

$= \boxed{\langle 1, \frac{2}{3}, \frac{7}{9}, \frac{20}{27}, \dots \rangle}$

② Write out the first four terms of the partial sum sequence for $\langle 0.3, 0.03, 0.003, 0.0003, \dots \rangle$.

$= \langle 0.3, 0.3+0.03, 0.3+0.03+0.003, 0.3+0.03+0.003+0.0003, \dots \rangle$

$= \boxed{\langle 0.3, 0.33, 0.333, 0.3333, \dots \rangle}$

③ Does $\sum_{n=2}^{\infty} \left(\frac{3}{2n} - \frac{3}{2n+2}\right)$ converge or diverge? If it converges, what is its value?

$= \lim_{n \to \infty} \left(\frac{3}{4} - \cancel{\frac{3}{6}}\right) + \left(\cancel{\frac{3}{6}} - \frac{3}{8}\right) + \cancel{\dots} + \left(\cancel{\frac{3}{2n}} - \frac{3}{2n+2}\right)$

$= \frac{3}{4} - \lim_{n \to \infty} \frac{3}{2n+2}$

$= \frac{3}{4} - 0 = \boxed{3/4}$ $\boxed{\text{Converges}}$

(4) Does $\sum_{j=2}^{\infty} \frac{6}{4j^2+4j}$ converge or diverge? If it converges, what is its value?

$$\frac{6}{4j(j+1)} = \frac{A}{4j} + \frac{B}{j+1}$$

$$6 = A(j+1) + B(4j)$$

Let $j=0$

$$6 = A(0+1) + \cancel{B(0)}$$

$$A = 6$$

Let $j=-1$

$$6 = \cancel{A(-1+1)} + B(4)(-1)$$

$$B = -3/2$$

$$= \sum_{j=2}^{\infty}\left(\frac{6}{4j} - \frac{3/2}{j+1}\right) = \sum_{j=2}^{\infty}\left(\frac{3}{2j} - \frac{3}{2j+2}\right)$$

$$= \boxed{3/4}$$ from problem (3)

$\boxed{\text{Converges}}$

(5) Compute $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$

$$= \left(-\frac{1}{3}\right)^0 + \left(-\frac{1}{3}\right)^1 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^3 + \cdots$$

$$= \sum_{n=0}^{\infty} \underset{\substack{\uparrow \\ a}}{(1)} \underset{\substack{\uparrow \\ r}}{\left(-\frac{1}{3}\right)}^n$$

Since $|r| = \left|-\frac{1}{3}\right| = \frac{1}{3} < 1$:

$$= \frac{a}{1-r} = \frac{1}{1-\left(-\frac{1}{3}\right)} = \frac{1}{4/3} = \boxed{3/4}.$$

(6) Prove $0.\overline{3} = 0.333\ldots$ equals $\frac{1}{3}$ using geo. series.

$$0.333\ldots = 0.3 + \frac{0.3}{10} + \frac{0.3}{100} + \frac{0.3}{1000} + \cdots$$

$$= \sum_{n=0}^{\infty} \underset{\substack{\uparrow \\ a}}{(0.3)} \underset{\substack{\uparrow \\ r}}{\left(\frac{1}{10}\right)}^n$$

Since $|r| = \left|\frac{1}{10}\right| = \frac{1}{10} < 1$:

$$= \frac{a}{1-r} = \frac{0.3}{1-0.1} = \frac{0.3}{0.9} = \boxed{\frac{1}{3}}.$$

⑦ Write $0.\overline{27} = 0.272727\ldots$ as a fraction of integers.

$$= 0.27 + 0.27\left(\frac{1}{100}\right) + 0.27\left(\frac{1}{10000}\right) + \cdots$$

$$= \sum_{n=0}^{\infty} 0.27\left(\frac{1}{100}\right)^n$$

$$= \frac{0.27}{1 - \frac{1}{100}} = \frac{27}{99} = \boxed{\frac{3}{11}}$$