MA 126 — Spring 2017 — Prof. Clontz — Standard Assessment 1

Name: Solutions

- Each question is prefaced with a Standard for this course.
- When grading, each response will be marked as follows:
  - ✓: The response is demonstrates complete understanding of the Standard.
  - $\star$: The response may indicate full understanding of the Standard, but clarification or minor corrections are required.
  - $\times$: The response does not demonstrate complete understanding of the Standard.
- Only responses marked with a ✓ mark count toward your grade for the semester. Visit the course website for more information on how to improve $\star$ and $\times$ marks.
- This Assessment is due after 50 minutes. All blank responses will be marked with $\times$.

C01: This student is able to... Derive properties of the logarithmic and exponential functions from their definitions.	Mark: (Instructor Use Only)

Use the definition $\ln x = \int_1^x \frac{1}{t}\,dt$ to prove that $\ln(x/a) = \ln x - \ln a$ for all positive real numbers x and a.

$$\frac{d}{dx}\left[\ln(x/a)\right] = \frac{d}{dx}\left[\ln\left(\frac{1}{a}x\right)\right] \qquad \frac{d}{dx}\left[\ln x - \ln a\right] = \frac{1}{x} - 0$$

$$= \frac{1}{x}$$

$$= \frac{1}{a}\left(\frac{1}{\frac{1}{a}x}\right)$$

$$= \frac{1}{\cancel{a}\,x} = \frac{1}{x}$$

Same derivative.

Thus $\ln(x/a) = \ln x - \ln a + C$ for all x.

Let $x = a$:

$$\ln(a/a) = \cancel{\ln a - \ln a} + C$$

$$\ln(1) = C$$

$$0 = C$$

Thus $\ln(x/a) = \ln x - \ln a$. $\square$

C02: This student is able to... Prove hyperbolic function identities.	Mark: (Instructor Use Only)

Use the definitions

$$\sinh(x) = \frac{e^x - e^{-x}}{2}, \cosh(x) = \frac{e^x + e^{-x}}{2}$$

to prove the following identity.

$$\cosh^2(x) = 1 + \sinh^2(x)$$

$$\cosh^2(x) = \left(\frac{e^x + e^{-x}}{2}\right)^2$$

$$= \frac{e^{2x} + 2\cancel{e^{x}e^{-x}} + e^{-2x}}{4}$$

$$= \frac{e^{2x} + e^{-2x}}{4} + \frac{1}{2}$$

$$1 + \sinh^2(x) = 1 + \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= 1 + \frac{e^{2x} - 2\cancel{e^{x}e^{-x}} + e^{-2x}}{4}$$

$$= 1 - \frac{1}{2} + \frac{e^{2x} + e^{-2x}}{4}$$

$$= \frac{1}{2} + \frac{e^{2x} + e^{-2x}}{4}$$

Thus

$\cosh^2(x) = 1 + \sinh^2(x)$.

$\square$

S01: This student is able to... Find derivatives and integrals involving logrithmic and exponential functions.	Mark: (Instructor Use Only)

a) Find $\frac{d}{dy}[\ln(y^2+1)+e^{3y}]$.

$$= \frac{1}{y^2+1}(2y) + e^{3y}(3)$$

$$= \boxed{\frac{2y}{y^2+1} + 3e^{3y}}$$

b) Find $\int \left(\frac{e}{x} + e^x\right)\, dx$.

$$= \int e\frac{1}{x} + e^x \, dx$$

$$= \boxed{e\ln|x| + e^x + C}$$

S02: This student is able to... Find derivatives and integrals involving hypberbolic functions.	Mark: (Instructor Use Only)

a) Find $\frac{d}{dx}[\cosh(2x-7) + \sinh\cancel{(x^2)\,\text{csch}}(x^2)]$.

$$= \frac{d}{dx}\left[\cosh(2x-7) + 1\right]$$

$$= \sinh(2x-7)(2) + 0$$

$$= \boxed{2\sinh(2x-7)}$$

OR

$$= \sinh(2x-7)(2) + \text{csch}(x^2)\cosh(x^2)(2x) + \cancel{\sinh(x^2)}(-\cancel{\text{csch}(x^2)}\coth(x^2))(2x)$$

$$= 2\sinh(2x-7) + \cancel{2x\coth(x^2)} - \cancel{2x\coth(x^2)}$$

$$= \boxed{2\sinh(2x-7)}$$

b) Find $\displaystyle\int 5\,\text{sech}(t)\tanh(t)\,dt$.

$$= 5(-\text{sech}(t)) + C$$

$$= \boxed{-5\,\text{sech}(t) + C}$$

Use this space if you need extra room for a problem: