MA 126 — Spring 2017 — Prof. Clontz — Standard Assessment 2

Name: Answers

- Each question is prefaced with a Standard for this course.
- When grading, each response will be marked as follows:
  - ✓: The response is demonstrates complete understanding of the Standard.
  - ⋆: The response may indicate full understanding of the Standard, but clarification or minor corrections are required.
  - ×: The response does not demonstrate complete understanding of the Standard.
- Only responses marked with a ✓ mark count toward your grade for the semester. Visit the course website for more information on how to improve ⋆ and × marks.
- This Assessment is due after 50 minutes. All blank responses will be marked with ×.

C01: This student is able to... Derive properties of the logarithmic and exponential functions from their definitions.	Mark: (Instructor Use Only)

Show that $5^2 = 25$ follows from the definition $a^x = \exp(x \ln(a))$. (Hint: Use the fact that $\ln(z) + \ln(z) = \ln(z \times z)$.)

$$5^2 = \exp(2\ln(5))$$
$$= \exp(\ln(5) + \ln(5))$$
$$= \cancel{\exp}(\cancel{\ln}(5 \cdot 5))$$
$$= 5 \cdot 5$$
$$= 25 \quad \square$$

C02: This student is able to... Prove hyperbolic function identities.	Mark: (Instructor Use Only)

Use the definition

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

to prove the following identity.

$$\cosh(2x) = 2\cosh^2(x) - 1$$

$$\cosh(2x) = \frac{e^{2x} + e^{-2x}}{2}$$

$$2\cosh^2(x) - 1 = 2\left(\frac{e^x + e^{-x}}{2}\right)^2 - 1$$

$$= \cancel{2}\left(\frac{e^{2x} + 2\cancel{e^x e^{-x}} + e^{-2x}}{\cancel{4}\,2}\right) - 1$$

$$= \frac{e^{2x} + \cancel{2} + e^{-2x}}{2} - \frac{\cancel{2}}{2}$$

$$= \frac{e^{2x} + e^{-2x}}{2}$$

Therefore $\cosh(2x) = 2\cosh^2(x) - 1$. $\square$

C03: This student is able to... Use integration by substitution.	Mark: (Instructor Use Only)

Show that $\int_2^3 x\sqrt{x-2}\,dx = \frac{26}{15}$.

$x = u + 2$

Let $u = x - 2$ $\qquad x = 3 \to u = 3 - 2 = 1$

$du = dx$ $\qquad x = 2 \to u = 2 - 2 = 0$

$$= \int_0^1 (u+2)\sqrt{u}\,du$$

$$= \int_0^1 \left(u^{3/2} + 2u^{1/2}\right)du$$

$$= \left[\frac{2}{5}u^{5/2} + \frac{4}{3}u^{3/2}\right]_0^1$$

$$= \left(\frac{2}{5} + \frac{4}{3}\right) - \cancel{(0+0)}$$

$$= \frac{6+20}{15} = \boxed{\frac{26}{15}}$$

S01: This student is able to... Find derivatives and integrals involving logrithmic and exponential functions.	Mark: (Instructor Use Only)

a) Find $\frac{d}{dz}[\ln(3e^z)]$.

$= \frac{1}{\cancel{3e^z}}(\cancel{3e^z})$

$= \boxed{1}$

OR

$= \frac{d}{dz}(\ln(3) + \cancel{\ln}(e^z))$

$= \frac{d}{dz}(\ln 3 + z)$

$= 0 + 1$

$= \boxed{1}$

b) Find $\int \left(2e + \frac{3}{y}\right)\, dy$.

$= \boxed{2ey + 3\ln|y| + C}$

S02: This student is able to... Find derivatives and integrals involving hypberbolic functions.	Mark: (Instructor Use Only)

a) Find $\frac{d}{dv}[4\tanh(3v) - \sinh(v^2)]$.

$$= 4\,\text{sech}^2(3v)(3) - \cosh(v^2)(2v)$$

$$= \boxed{12\,\text{sech}^2(3v) - 2v\cosh(v^2)}$$

b) Find $\int (\cosh(x) + 2\sinh(x))\,dx$.

$$= \boxed{\sinh(x) + 2\cosh(x) + C}$$

S03: This student is able to... Integrate products of trigonometric functions.	Mark: (Instructor Use Only)

Find $\int \sin^3(\theta)\cos^3(\theta)\,d\theta$.

$$= \int \sin^2\theta \cos^3\theta \sin\theta \, d\theta$$
$$= \int (1-\cos^2\theta)\cos^3\theta \sin\theta \, d\theta$$

Let $u = \cos\theta$

$-du = +\sin\theta \, d\theta$

$$= \int (1-u^2)u^3(-du)$$
$$= \int (u^5 - u^3)\,du$$
$$= \frac{1}{6}u^6 - \frac{1}{4}u^4 + C$$
$$= \boxed{\frac{1}{6}\cos^6\theta - \frac{1}{4}\cos^4\theta + C}$$

OR

$$= \int \sin^3\theta \cos^2\theta \cos\theta \, d\theta$$
$$= \int \sin^3\theta(1-\sin^2\theta)\cos\theta \, d\theta$$

Let $u = \sin\theta$

$du = \cos\theta \, d\theta$

$$= \int u^3(1-u^2)\,du$$
$$= \int (u^3 - u^5)\,du$$
$$= \frac{1}{4}u^4 - \frac{1}{6}u^6 + C$$
$$= \boxed{\frac{1}{4}\sin^4\theta - \frac{1}{6}\sin^6\theta + C}$$

S04: This student is able to... Use trigonometric substitution.	Mark: (Instructor Use Only)

Find $\int \frac{2}{1+4x^2}\,dx$.

Let $1+4x^2 = 1+\tan^2\theta = \sec^2\theta$

$4x^2 = \tan^2\theta$

$2x = \tan\theta \Rightarrow \theta = \tan^{-1}(2x)$

$2dx = \sec^2\theta\, d\theta$

$= \int \frac{1}{\cancel{\sec^2\theta}} \cancel{\sec^2\theta}\, d\theta$

$= \int d\theta$

$= \theta + C$

$= \boxed{\tan^{-1}(2x) + C}$

OR

$= 2\int \frac{1}{1+4x^2}\,dx$

Let $4x^2 = u^2$

$2x = u$

$2dx = du$

$= \int \frac{1}{1+u^2}\,du$

$= \tan^{-1}(u) + C$

$= \boxed{\tan^{-1}(2x) + C}$

S05: This student is able to... Use partial fractions to integrate rational functions.	Mark: (Instructor Use Only)

a) Complete the following partial fraction expansion:

$$\frac{f(x)}{(x+3)^3(x^2+7)^2} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3} + \frac{Dx+E}{x^2+7} + \frac{Fx+G}{(x^2+7)^2}$$

(Assume the degree of f is less than 7. You do NOT need to solve for A through G.)

b) Find $\int \frac{8x^2-6x+14}{(x-1)(x^2+7)}\,dx$.

$$\frac{8x^2-6x+14}{(x-1)(x^2+7)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+7}$$

$$8x^2-6x+14 = A(x^2+7) + (Bx+C)(x-1)$$

Let $x=1$

$$8-6+14 = A(8) + \cancel{(Bx+C)(1-1)}$$

$$16 = 8A$$

$$2 = A$$

$$8x^2-6x+14 = 2x^2+14 + Bx^2+Cx-Bx+C$$

$$6x^2-6x = Bx^2+(C-B)x+C$$

x^2 | $6=B$

Const | $0=C$

$$= \int\left(\frac{2}{x-1} + \frac{6x}{x^2+7}\right)dx = \boxed{2\ln|x-1| + 3\ln|x^2+7| + C}$$

Use this space if you need extra room for a problem: