MA 126 — Spring 2017 — Prof. Clontz — Standard Assessment ~~2~~ 3

Name: Answers

- Each question is prefaced with a Standard for this course.
- When grading, each response will be marked as follows:
  - ✓: The response is demonstrates complete understanding of the Standard.
  - ⋆: The response may indicate full understanding of the Standard, but clarification or minor corrections are required.
  - ×: The response does not demonstrate complete understanding of the Standard.
- Only responses marked with a ✓ mark count toward your grade for the semester. Visit the course website for more information on how to improve ⋆ and × marks.
- This Assessment is due after 50 minutes. All blank responses will be marked with ×.

C01: This student is able to... Derive properties of the logarithmic and exponential functions from their definitions.	Mark: (Instructor Use Only)

Let $f^{\leftarrow}$ denote the inverse function of an invertable function f; in particular, if $f(x) = \ln(x)$, then $f^{\leftarrow}(x) = \exp(x)$.
Use the theorem $\frac{d}{dx}[f^{\leftarrow}(x)] = \frac{1}{f'(f^{\leftarrow}(x))}$ to prove that $\frac{d}{dx}[\exp x] = \exp x$.

$f'(x) = \frac{1}{x}$

$$\frac{d}{dx}\left[\exp(x)\right] = \frac{d}{dx}\left[f^{\leftarrow}(x)\right]$$

$$= \frac{1}{f'(f^{\leftarrow}(x))}$$

$$= \frac{1}{1/\exp(x)}$$

$$= \exp(x) \quad \square$$

C02: This student is able to... Prove hyperbolic function identities.	Mark: (Instructor Use Only)

Use the definitions

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}, \operatorname{sech}(x) = \frac{2}{e^x + e^{-x}}$$

to prove the following identity.

$$1 - \tanh^2(x) = \operatorname{sech}^2(x)$$

$$1 - \tanh^2(x) = 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2$$

$$= 1 - \frac{e^{2x} - 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}}$$

$$= \frac{(\cancel{e^{2x}} + 2 + \cancel{e^{-2x}}) - (\cancel{e^{2x}} - 2 + \cancel{e^{-2x}})}{e^{2x} + 2 + e^{-2x}}$$

$$= \frac{4}{e^{2x} + 2 + e^{-2x}}$$

$$\operatorname{sech}^2(x) = \left(\frac{2}{e^x + e^{-x}}\right)^2$$

$$= \frac{4}{e^{2x} + 2 + e^{-2x}}$$

Thus $1 - \tanh^2(x) = \operatorname{sech}^2(x)$. $\square$

C03: This student is able to... Use integration by substitution.	Mark: (Instructor Use Only)

Recall that $\frac{d}{du}[\tan u] = \sec^2 u$. Find $\int 4x \sec^2(x^2)\, dx$.

Let $u = x^2$

$du = 2x\,dx$

$2du = 4x\,dx$

$= \int 2 \sec^2(u)\,du$

$= 2\tan(u) + C$

$= \boxed{2\tan(x^2) + C}$

C04: This student is able to... Use integration by parts.	Mark: (Instructor Use Only)

Find $\int r^2 e^r \, dr$.

Let $u = r^2$ $\quad v = e^r$
$du = 2r\,dr$ $\quad dv = e^r dr$

$= r^2 e^r - \int 2re^r dr$

Let $u = 2r$ $\quad v = e^r$
$du = 2dr$ $\quad dv = e^r dr$

$= r^2 e^r - \left[2re^r - \int 2e^r dr\right]$

$= r^2 e^r - 2re^r + \int 2e^r dr$

$= \boxed{r^2 e^r - 2re^r + 2e^r + C}$

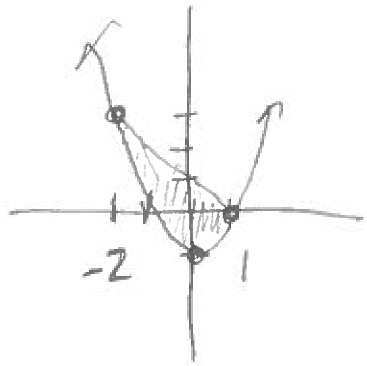
C05: This student is able to... Identify and use appropriate integration techniques.	Mark: (Instructor Use Only)

Draw lines matching each of the five integrals on the left with the most appropriate integration technique listed on the right. Multiple techniques may be technically possible, but choose the technique most useful to begin integration. Every integral and technique is used exactly once in the correct answer.

$\int x\sin(x)\,dx$	• Integration by Substiution
$\int \cos^4(x)\,dx$	• Method of Partial Fractions
$\int \frac{1}{x\sqrt{x^2+1}}\,dx$	• Trigonometric Identities
$\int \frac{4x}{3x^2-1}\,dx$	• Trigonometric Substitution
$\int \frac{7x^2+x+12}{x^3+3x}\,dx$	• Integration by Parts

C06: This student is able to... Express an area between curves as a definite integral.	Mark: (Instructor Use Only)

Find a definite integral equal to the area between the curves $y = 1 - x$ and $y = x^2 - 1$.



$1-x = x^2-1$

$0 = x^2+x-2$

$0 = (x+2)(x-1)$

x	$y=1-x$	$y=x^2-1$
-2	3	3
-1	2	0
0	1	-1
1	0	0

$$A = \boxed{\int_{-2}^{1} \left((1-x)-(x^2-1)\right) dx}$$

S03: This student is able to... Integrate products of trigonometric functions.	Mark: (Instructor Use Only)

Find $\int 2\cos^2(y)\,dy$.

$$= \int 2\left(\tfrac{1}{2}+\tfrac{1}{2}\cos(2y)\right)dy$$

$$= \int \left(1+\cos(2y)\right)dy$$

$$= \boxed{y+\tfrac{1}{2}\sin(2y)+C}$$

S04: This student is able to... Use trigonometric substitution.	Mark: (Instructor Use Only)

Find $\int \frac{z+1}{\sqrt{1-z^2}}\,dz$.

Let $1-z^2 = 1-\sin^2\theta = \cos^2\theta$

$z^2 = \sin^2\theta$ $\qquad \cos\theta = \sqrt{1-z^2}$

$z = \sin\theta$

$dz = \cos\theta\, d\theta \qquad \theta = \sin^{-1}(z)$

$= \int \frac{\sin\theta + 1}{\cancel{\sqrt{\cos^2\theta}}} \cancel{\cos\theta}\, d\theta$

$= \int (\sin\theta + 1)\, d\theta$

$= -\cos\theta + \theta + C$

$= \boxed{-\sqrt{1-z^2} + \sin^{-1}(z) + C}$

S05: This student is able to... Use partial fractions to integrate rational functions.	Mark: (Instructor Use Only)

a) Complete the following partial fraction expansion:

$$\frac{f(x)}{(x^2+3)^2(x-7)^4} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{(x^2+3)^2} + \frac{E}{x-7} + \frac{F}{(x-7)^2} + \frac{G}{(x-7)^3} + \frac{H}{(x-7)^4}$$

(Assume the degree of f is less than 8. You do NOT need to solve for your constants.)

b) Find $\displaystyle\int \frac{3x^2-x+2}{(x^2+1)(x-1)}\,dx$.

$$\frac{3x^2-x+2}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$$

$$3x^2-x+2 = (Ax+B)(x-1) + C(x^2+1)$$

Let $x=1$

$$3-1+2 = 0 + C(2)$$
$$4 = 2C$$
$$2 = C$$

$$3x^2-x+\cancel{2} = (Ax+B)(x-1) + 2x^2 + \cancel{2}$$
$$x^2 - x = (Ax+B)(x-1)$$

Let $x=0$

$$0 = (B)(-1)$$
$$B = 0$$

Let $x=2$

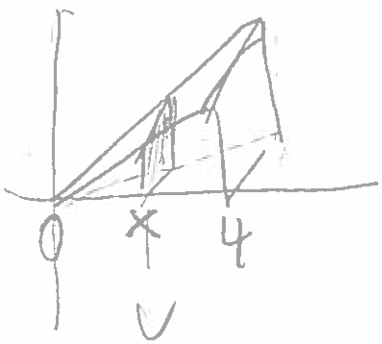
$$4-2 = (A(2)+0)(\cancel{2-1})$$
$$2 = 2A$$
$$1 = A$$

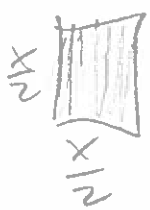
$$= \int \left(\frac{x}{x^2+1} + \frac{2}{x-1}\right) dx$$

$$= \boxed{\frac{1}{2}\ln(x^2+1) + 2\ln|x-1| + C}$$

S06: This student is able to... Use cross-sectioning to express a volume as a definite integral.	Mark: (Instructor Use Only)

Find a definite integral that equals the volume of a pyramid positioned from $x = 0$ to $x = 4$ with square cross-sections of side length $\frac{x}{2}$.





$$A(x) = \left(\frac{x}{2}\right)^2$$

$$= \frac{x^2}{4}$$

$$V = \boxed{\int_0^4 \frac{x^2}{4}\,dx}$$