MA 126 — Spring 2017 — Prof. Clontz — Standard Assessment 4

Name: Answers

- Each question is prefaced with a Standard for this course.
- When grading, each response will be marked as follows:
  - ✓: The response is demonstrates complete understanding of the Standard.
  - ⋆: The response may indicate full understanding of the Standard, but clarification or minor corrections are required.
  - ×: The response does not demonstrate complete understanding of the Standard.
- Only responses marked with a ✓ mark count toward your grade for the semester. Visit the course website for more information on how to improve ⋆ and × marks.
- This Assessment is due after 50 minutes. All blank responses will be marked with ×.

C03: This student is able to... Use integration by substitution.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Find $\int \frac{6x^2+14}{x^3+7x-3}\,dx$.

Let $u = x^3+7x-3$

$du = (3x^2+7)\,dx$

$2du = (6x^2+14)\,dx$

$= \int \frac{2}{u}\,du$

$= 2\ln|u| + C$

$= \boxed{2\ln|x^3+7x-3| + C}$

	Mark:	Reattempt/ Correction:
C04: This student is able to... Use integration by parts.	(Instructor Use Only)	(Instructor Use Only)

Find $\int 8e^y \sin(y)\, dy$.

Let $u = 8e^y$ $v = -\cos(y)$

$du = 8e^y dy$ $dv = \sin(y)\, dy$

$= -8e^y \cos(y) + \int 8e^y \cos(y)\, dy$

Let $u = 8e^y$ $v = \sin(y)$

$du = 8e^y dy$ $dv = \cos(y)\, dy$

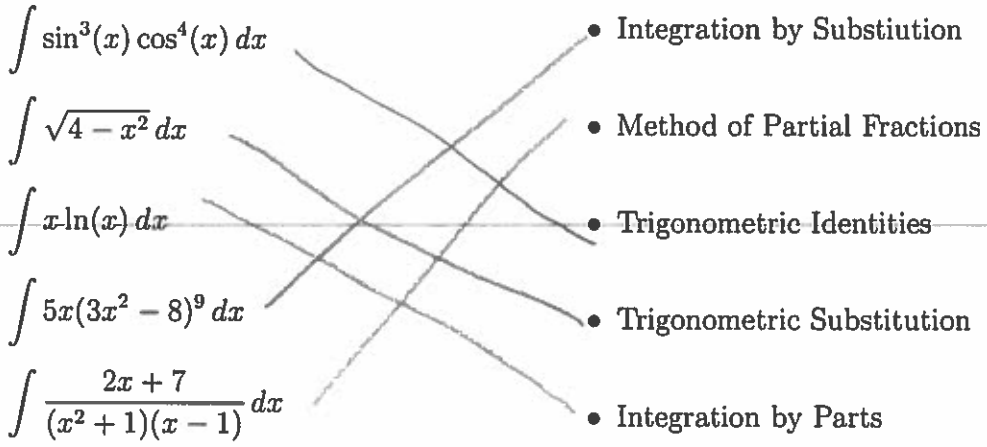
$\int 8e^y \sin(y)\, dy = -8e^y \cos(y) + 8e^y \sin(y) - \int 8e^y \sin(y)\, dy$

$2\int 8e^y \sin(y)\, dy = -8e^y \cos(y) + 8e^y \sin(y) + C$

$\int 8e^y \sin(y)\, dy = \boxed{-4(e^y \cos(y) + 4e^y \sin(y) + C}$

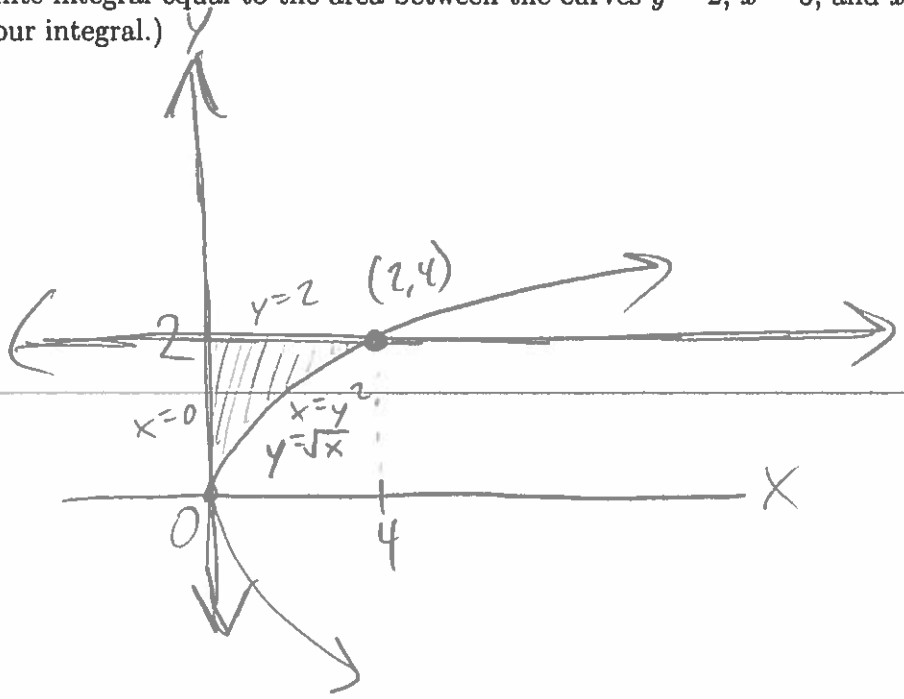
C05: This student is able to... Identify and use appropriate integration techniques.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Draw lines matching each of the five integrals on the left with the most appropriate integration technique listed on the right. Multiple techniques may be technically possible, but choose the technique most useful to begin integration. Every integral and technique is used exactly once in the correct answer.



C06: This student is able to... Express an area between curves as a definite integral.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Find a definite integral equal to the area between the curves $y = 2$, $x = 0$, and $x = y^2$. (Do not solve your integral.)

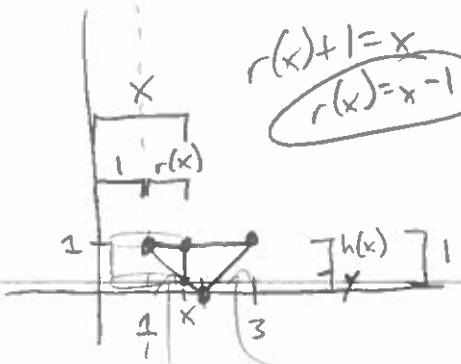


$$A = \int_0^4 \left((2) - (\sqrt{x})\right) dx \quad \text{OR} \quad A = \int_0^2 \left((y^2) - (0)\right) dy$$

C07: This student is able to... Use the washer or cylindrical shell method to express a volume of revolution as a definite integral.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Find a definite integral equal to the volume of the solid of revolution obtained by rotating the triangle with vertices $(1,1)$, $(2,0)$, and $(3,1)$ around the axis $x = 1$. (Do not solve your integral.)

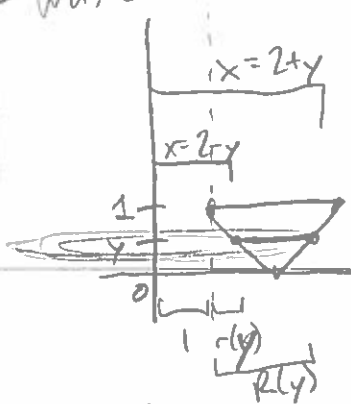
Cyl. Shell



$r(x)+1=x$

$r(x)=x-1$

OR Washer



$r(y)+1=2-y$

$r(y)=1-y$

$R(y)+1=2+y$

$R(y)=1+y$

$y=2-x$ between 1 and 2

$h(x)+y=1$

$h(x)+2-x=1$

$h(x)=x-1$

$y=x-2$ between 2 and 3

$h(x)+y=1$

$h(x)+x-2=1$

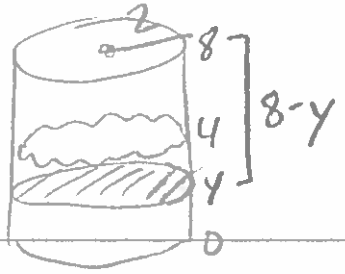
$h(x)=3-x$

$$V=\pi\int_0^1\left((1+y)^2-(1-y)^2\right)dy$$

$$V=2\pi\int_1^2(x-1)(x-1)\,dx+2\pi\int_2^3(x-1)(3-x)\,dx$$

C08: This student is able to... Express the work done in a system as a definite integral.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Find a definite integral equal to the work (in kilojoules) required to pump the contents of a cylindrical tank with a radius of 2 meters and a total height of 8 meters to its top, assuming it is initially half-full of salt water that weighs 10 kilonetwons per square meter. (Do not solve your integral.)

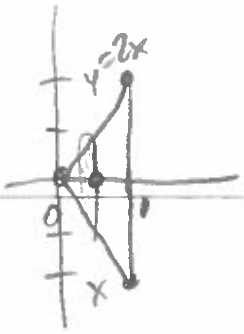


$$A = \pi(2)^2$$
$$= 4\pi$$
$$dV = 4\pi\,dy$$
$$dF = 40\pi\,dy$$
$$dW = 40\pi(8-y)\,dy$$

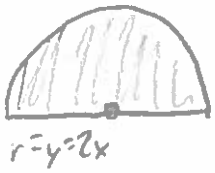
$$W = \int_0^4 40\pi(8-y)\,dy$$

S06: This student is able to... Use cross-sectioning to express a volume as a definite integral.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Find a definite integral that equals the volume of a solid whose base is the triangle with vertices $(0,0)$, $(1,2)$, and $(1,-2)$, and whose cross-sections perpindicular to the x-axis are semicircles with diameters on the xy plane. (Do not solve your integral.)



$$V = \int_0^1 4\pi x^2 \, dx$$

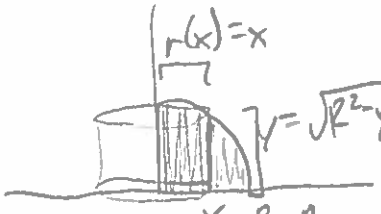


$$A(x) = \pi r^2$$
$$= \pi(2x)^2$$
$$= 4\pi x^2$$

S07: This student is able to... Derive a formula for the volume of a three dimensional solid.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Prove that the volume of a hemisphere (~~half-sphere~~) with radius R is given by $V = \frac{2}{3}\pi R^3$.

(Shell Method)



$$V = 2\pi \int_0^R r(x)\,h(x)\,dx$$

$$V = 2\pi \int_0^R x\sqrt{R^2-x^2}\,dx$$

$$= 2\pi \left[-\frac{1}{3}(R^2-x^2)^{3/2}\right]_0^R$$

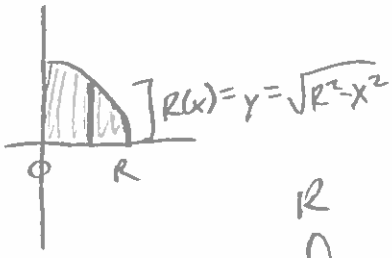
$$= 2\pi \left[\cancel{-\frac{1}{3}(0)^{3/2}} + \frac{1}{3}(R^2)^{3/2}\right]$$

$$V = \frac{2}{3}\pi R^3 \quad \square$$

OR

(Washer Method)

$(r(x)=0)$



$$V = \pi \int_0^R R(x)^2\,dx$$

$$= \pi \int_0^R (R^2-x^2)\,dx$$

$$= \pi \left[R^2x - \frac{1}{3}x^3\right]_0^R$$

$$= \pi \left[R^3 - \frac{1}{3}R^3\right]$$

$$= \pi \left[\frac{2}{3}R^3\right]$$

$$V = \frac{2}{3}\pi R^3 \quad \square$$

S08: This student is able to... Parametrize planar curves and sketch parametrized curves.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

a) Give a parameterization of the parabola $y = x^2$ from $(-1, 1)$ to $(3, 9)$.

$x = t$

$y = x^2$

$y = t^2$

$-1 \leq x \leq 3$

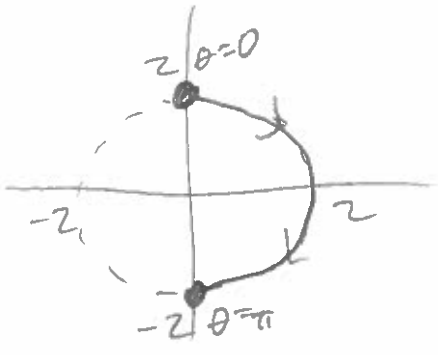
$-1 \leq t \leq 3$

b) Sketch the curve parameterized by $x = 2\sin\theta$, $y = 2\cos\theta$ for $0 \leq \theta \leq \pi$.

(clockwise)

$x^2 + y^2 = 4\sin^2\theta + 4\cos^2\theta$

$x^2 + y^2 = 4$



At $\theta = 0$

$x = 2\sin 0$ $\quad y = 2\cos 0$

$x = 0$ $\quad y = 2$

At $\theta = \pi$

$x = 2\sin\pi$ $\quad y = 2\cos\pi$

$x = 0$ $\quad y = -2$