MA 126 — Spring 2017 — Prof. Clontz — Standard Assessment 5

Name: Answers

- Each question is prefaced with a Standard for this course.
- When grading, each response will be marked as follows:
  - ✓: The response is demonstrates complete understanding of the Standard.
  - $\star$: The response may indicate full understanding of the Standard, but clarification or minor corrections are required.
  - $\times$: The response does not demonstrate complete understanding of the Standard.
- Only responses marked with a ✓ mark count toward your grade for the semester. Visit the course website for more information on how to improve $\star$ and $\times$ marks.
- This Assessment is due after 50 minutes. All blank responses will be marked with $\times$.

C04: This student is able to... Use integration by parts.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Find $\int 3x\cosh(x)\,dy$.

Let $u = 3x$ $\quad v = \sinh(x)$

$du = 3dx$ $\quad dv = \cosh(x)$

$= 3x\sinh(x) - \int 3\sinh(x)dx$

$= \boxed{3x\sinh(x) - 3\cosh(x) + C}$

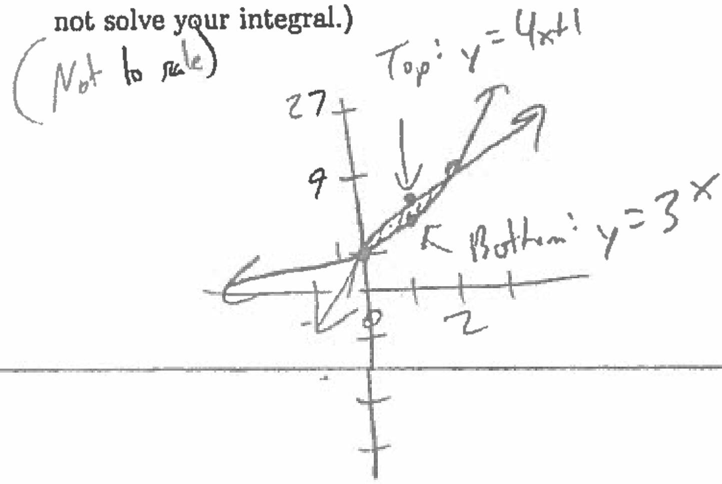
C05: This student is able to... Identify and use appropriate integration techniques.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Draw lines matching each of the five integrals on the left with the most appropriate integration technique listed on the right. Multiple techniques may be technically possible, but choose the technique most useful to begin integration. Every integral and technique is used exactly once in the correct answer.

$\int 8x^3 \ln(x^4+7)\,dx$	• Integration by Substiution
$\int 8\sec^3(x)\tan^5(x)\,dx$	• Method of Partial Fractions
$\int \frac{1+3x}{x^3-x}\,dx$	• Trigonometric Identities
$\int 2\sin(x)\cosh(x)\,dx$	• Trigonometric Substitution
$\int \frac{1}{\sqrt{4x^2+1}}\,dx$	• Integration by Parts

C06: This student is able to... Express an area between curves as a definite integral.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Find a definite integral equal to the area between the curves $y = 3^x$ and $y = 4x + 1$. (Do not solve your integral.)



x	3^x	$4x+1$
-1	$1/3$	-3
0	1	1
1	3	5
2	9	9
3	27	13

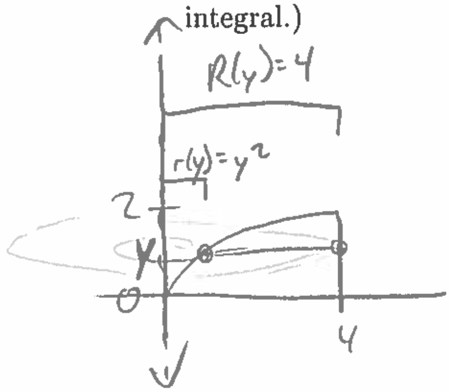
$$A = \boxed{\int_0^2 \left((4x+1) - (3^x)\right) dx}$$

C07: This student is able to... Use the washer or cylindrical shell method to express a volume of revolution as a definite integral.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Washer

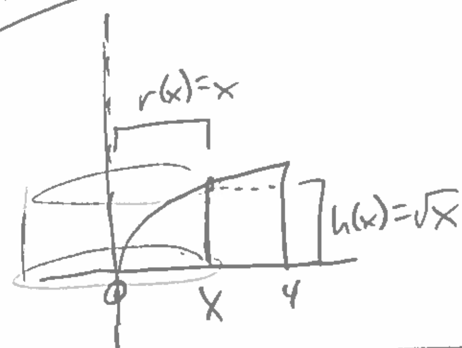
Find a definite integral equal to the volume of the solid of revolution obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ around the axis $x = 0$. (Do not solve your integral.)

$x = y^2$



$$V = \boxed{\pi \int_0^2 \left((4)^2 - (y^2)^2 \right) dy}$$

Cyl Shell



$$V = \boxed{2\pi \int_0^4 (x)(\sqrt{x})\, dx}$$

	Mark:	Reattempt/ Correction:
C08: This student is able to... Express the work done in a system as a definite integral.	(Instructor Use Only)	(Instructor Use Only)

Find a definite integral equal to the work (in foot-pounds) required to pull out a 100-foot 25-pound rope that is fully extended into a well. (Do not solve your integral.)

$$F(x) = \underset{\text{feet}}{(100-x)}\underset{\text{lbs per ft}}{\left(\frac{25}{100}\right)} = 25 - \frac{1}{4}x$$

$$\boxed{W = \int_0^{100} \left(25 - \frac{1}{4}x\right) dx}$$

	Mark:	Reattempt/ Correction:
C09: This student is able to... Parametrize a curve to express an arclength or area as a definite integral.	(Instructor Use Only)	(Instructor Use Only)

Find a definite integral equal to the arclength of portion of the parabola $x = y^2$ between the points $(1, 1)$ and $(4, -2)$. (Do not solve your integral.)

Let $y=t$
$x=t^2$ $\quad -2 \leq t \leq 1$

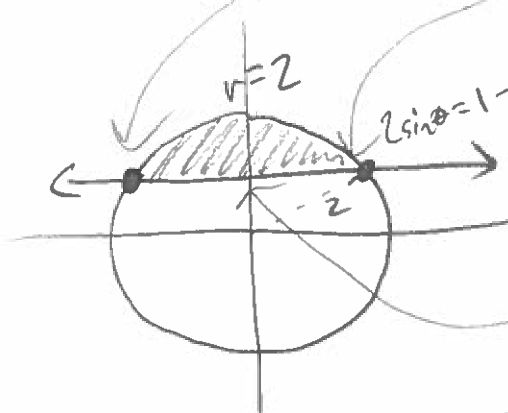
$$L = \int_{-2}^{1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \boxed{\int_{-2}^{1} \sqrt{(2t)^2 + (1)^2}\, dt}$$

C10: This student is able to... Use polar coordinates to express an arclength or area as a definite integral.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Find a definite integral equal to the area inside the circle $x^2 + y^2 = 4$ and above the line $y = 1$. (Do not solve your integral.)

$2\sin\theta = 1 \rightarrow \sin\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$



$y = 1$

$r\sin\theta = 1$

$r = \frac{1}{\sin\theta}$

$$A = \boxed{\frac{1}{2}\int_{\pi/6}^{5\pi/6}\left((2)^2 - \left(\frac{1}{\sin\theta}\right)^2\right)d\theta}$$

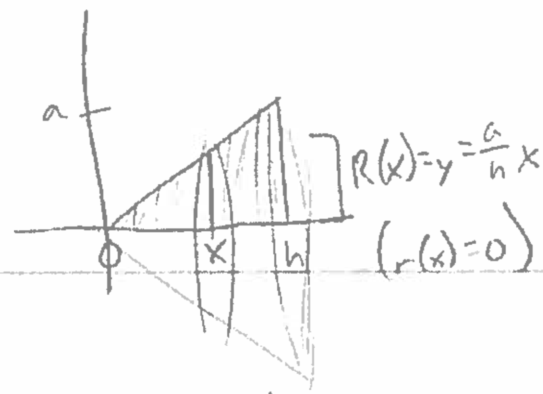
C11: This student is able to... Compute the limit of a convergent sequence.	Mark:	Reattempt/ Correction:

Find $\lim_{n\to\infty} \frac{\sin(n)+1}{n^2}$.

$$\boxed{0} = \lim_{n\to\infty} \frac{-1+1}{n^2} \leq \lim_{n\to\infty} \frac{\sin(n)+1}{n^2} \leq \lim_{n\to\infty} \frac{1+1}{n^2} = \boxed{0}$$

S07: This student is able to... Derive a formula for the volume of a three dimensional solid.	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

Prove that the volume of a cone with radius a and height h is $V = \frac{1}{3}\pi a^2 h$. (Hint: Start by letting $y = \frac{a}{h}x$ be the hypotenuse of a right triangle with legs length h and a.)



$$V = \pi\int_0^h \left(\left(\frac{a}{h}x\right)^2 - (0)^2\right)dx$$

$$= \pi\frac{a^2}{h^2}\int_0^h x^2\,dx$$

$$= \frac{1}{3}\pi\frac{a^2}{h^2}\left[x^3\right]_0^h$$

$$= \frac{1}{3}\pi\frac{a^2}{\cancel{h^2}}h^{\cancel{3}}$$

$$= \frac{1}{3}\pi a^2 h \quad \square$$

S08: This student is able to... Parametrize planar curves and sketch parametrized curves.	Mark: (Instructor Use Only)	Reattempt / Correction: (Instructor Use Only)

a) Give a parameterization of the line segment with endpoints $(4,3)$ and $(-1,2)$.

$x = x_0 + (x_1 - x_0)t$
$y = y_0 + (y_1 - y_0)t$

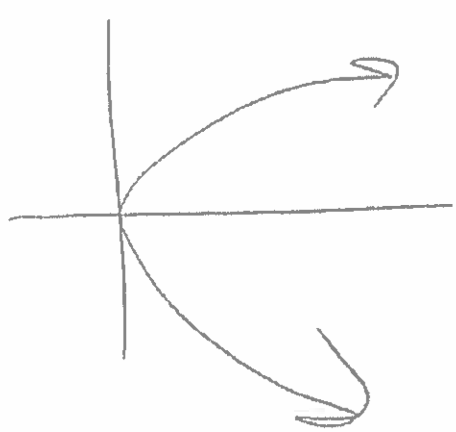
$x = 4 + (-1-4)t$
$y = 3 + (2-3)t$

$\boxed{\begin{array}{l} x = 4 - 5t \\ y = 3 - t \end{array}}$

$\boxed{0 \le t \le 1}$

b) Sketch the curve parameterized by $x = 4t^2$, $y = 2t$ for all real numbers t.

$x = (2t)^2$
$x = y^2$



S09: This student is able to... **Use parametric equations to find and use tangent slopes.**	Mark: (Instructor Use Only)	Reattempt/ Correction: (Instructor Use Only)

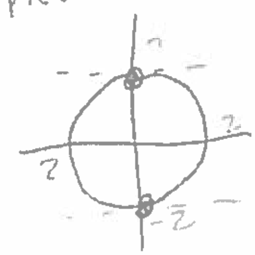
Find the points on the parametric curve defined by $x = 3\sin t$, $y = -2\cos t$ for $0 \le t \le 2\pi$ that have horizontal tangent lines.

$\frac{dy}{dx} = 0$

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

$$\frac{dy}{dt} = 0$$

Picture:



$$2\sin t = 0$$

$$\sin t = 0$$

$$t = 0, \pi, \cancel{2\pi} \text{ (same as } 0\text{)}$$

$x(0) = 3\sin 0 = 0$ $\quad$ $y(0) = -2\cos 0 = -2$ $\quad$ $\boxed{(0,-2)}$

$x(\pi) = 3\sin\pi = 0$ $\quad$ $y(\pi) = -2\cos\pi = +2$ $\quad$ $\boxed{(0,2)}$

S10: This student is able to... Convert and sketch polar and Cartesian coordinates and equations.	Mark:	Reattempt/ Correction:

a) Find a polar coordinate equal to the Cartesian coordinate $(1, -\sqrt{3})$.

$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{1} = \frac{-\sqrt{3}/2}{1/2} = \frac{\sin\theta}{\cos\theta}$$

$$r = \sqrt{x^2+y^2} = \sqrt{1+3} = \sqrt{4} = 2$$

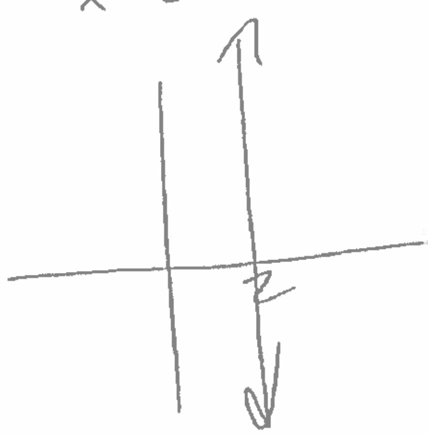
$\theta = -\pi/3$ OR $5\pi/3$

$\boxed{p(2, -\pi/3)}$ OR $\boxed{p(2, 5\pi/3)}$

b) Sketch the polar curve $r = 2\sec\theta$ in the xy plane.

$r\cos\theta = 2$

$x = 2$



S11: This student is able to... Define and use explicit and recursive formulas for sequences.	Mark:	Reattempt/ Correction:

Prove that $a_0 = 1$, $a_{n+1} = 3a_n$ is a recursive formula for the sequence defined explicitly by $a_n = 3^n$.

$a_0 = 3^0 = 1$ ✓

$a_{n+1} = 3^{n+1}$

$= 3^1 3^n$

$= 3a_n$ ✓