

Module S: Structure of Vector Spaces

Readiness Assurance Test

Choose the most appropriate response for each question.

1) Simplify the following Euclidean vector expression.

$$4 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix}$

(b) $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

2) Express the following system of linear equations as an augmented matrix.

$$\begin{aligned} 2x_1 + x_2 + 4x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \\ -3x_1 + 4x_2 + x_3 &= -7 \end{aligned}$$

(a) $\left[\begin{array}{ccc|c} 2 & 1 & 4 & 0 \\ 1 & 1 & 1 & 1 \\ -3 & 4 & 1 & -7 \end{array} \right]$

(b) $\left[\begin{array}{ccc|c} 2 & 1 & -3 & -3 \\ 1 & 1 & 4 & 4 \\ 4 & 1 & 1 & 1 \\ 0 & 1 & -7 & -7 \end{array} \right]$

(c) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -2 & 4 & 4 \\ 4 & 1 & 1 & 1 \\ 0 & 1 & -7 & -7 \end{array} \right]$

(d) $\left[\begin{array}{ccc|c} 2 & 1 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ -3 & 4 & 1 & -7 \end{array} \right]$

3) Find RREF $\left[\begin{array}{ccc} 1 & 2 & 3 \\ 3 & 2 & 5 \\ -2 & 0 & -2 \end{array} \right]$.

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4) Find the solution set of the following system of linear equations.

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4 \\ 2x_1 + 3x_2 + x_4 &= 0 \end{aligned}$$

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

(c) $\left\{ \left[\begin{array}{c} 1 \\ 1 + 3a \\ 4 + a \\ -5 + a \end{array} \right] \middle| a \in \mathbb{R} \right\}$

(b) $\left\{ \left[\begin{array}{c} 12 - 3a - 2b \\ -8 + 2a + b \\ a \\ b \end{array} \right] \middle| a, b \in \mathbb{R} \right\}$

(d) The system is inconsistent

5) Find the solution set of the following system of linear equations.

$$\begin{aligned} 2x_1 + x_2 + 4x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \\ -3x_1 + 4x_2 + x_3 &= -7 \end{aligned}$$

- (a) $\left\{ \begin{bmatrix} 1 \\ -6 \\ 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1+a \\ -2 \\ 1+3a \end{bmatrix} \mid a \in \mathbb{R} \right\}$ (c) $\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \right\}$ (d) The system is inconsistent

6) How many vectors are required to span all of \mathbb{R}^4 (the space of Euclidean vectors with four components)?

- (a) Infinitely Many (b) 3 (c) 4 (d) 5

7) How many vectors are required to span all of \mathcal{P}^3 (the space of polynomials of degree three or less)?

- (a) Infinitely Many (b) 2 (c) 3 (d) 4

8) Which vector is a linear combination of $\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$?

- (a) $\begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 3 \\ -7 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} -5 \\ 3 \\ 1 \\ 1 \end{bmatrix}$

9) Which vector belongs to $\text{span} \left\{ \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$?

- (a) $\begin{bmatrix} 3 \\ -7 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \end{bmatrix}$

10) What best describes $\text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ in three-dimensional Euclidean space \mathbb{R}^3 ?

- (a) all of \mathbb{R}^3 (b) a line (c) a plane (d) a sphere