Module A: Algebraic properties of linear maps

Readiness Assurance Test

Choose the most appropriate response for each question.

1) Which of the following is a solution to the system of linear equations

$$\begin{aligned} x+3y-z&=2\\ 2x+8y+3z&=-1\\ -x-y+9z&=-10 \end{aligned}$$

(a) $\begin{bmatrix}1\\1\\0\end{bmatrix}$ (b) $\begin{bmatrix}1\\0\\-1\end{bmatrix}$ (c) $\begin{bmatrix}0\\1\\-1\end{bmatrix}$ (d) $\begin{bmatrix}1\\-1\\1\end{bmatrix}$

2) Find a basis for the solution set of the following homogeneous system of linear equations

$$\begin{aligned} x+2y+-z-w&=0\\ -2x-4y+3z+5w&=0 \end{aligned}$$

(a) $\left\{\begin{bmatrix}-2\\1\\0\\0\end{bmatrix},\begin{bmatrix}-2\\0\\-3\\1\end{bmatrix}\right\}$ (b) $\left\{\begin{bmatrix}1\\2\\0\\2\end{bmatrix},\begin{bmatrix}0\\0\\1\\3\end{bmatrix}\right\}$ (c) $\left\{\begin{bmatrix}2\\-1\\3\\-1\end{bmatrix}\right\}$ (d) $\left\{\begin{bmatrix}1\\2\\1\\5\end{bmatrix}\right\}$

3) Determine which property applies to the set of vectors

$$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix}\right\}\subseteq\mathbb{R}^3.$$

(a) It is a basis of $\mathbb{R}^3$.
(b) It spans but it is linearly dependent
(c) It does not span and is linearly independent
(d) It does not span and is linearly dependent

4) Determine which property applies to the set of vectors

$$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}2\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\3\end{bmatrix}\right\}\subseteq\mathbb{R}^3.$$

(a) It does not span and is linearly independent
(b) It does not span and is linearly dependent
(c) It is a basis of $\mathbb{R}^3$.
(d) It spans but it is linearly dependent

5) Determine which property applies to the set of vectors

$$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}-2\\0\\-2\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}3\\3\\-3\end{bmatrix}\right\}\subseteq\mathbb{R}^3.$$

(a) It is a basis of $\mathbb{R}^3$.
(b) It spans but it is linearly dependent
(c) It does not span and is linearly dependent
(d) It does not span and is linearly independent

6) Determine which property applies to the set of vectors

$$\left\{ \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ -4 \end{bmatrix} \right\} \subseteq \mathbb{R}^3.$$

(a) It spans but it is linearly dependent
(b) It is a basis of $\mathbb{R}^3$.
(c) It does not span and is linearly independent
(d) It does not span and is linearly dependent

7) Suppose $\mathbf{v}_1, \ldots, \mathbf{v}_n \in \mathbb{R}^5$ and you know that every vector in span $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ can be written uniquely as a linear combination of the vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$. What can you conclude about n?

(a) $n \geq 5$
(b) $n \leq 5$
(c) $n = 5$
(d) n could be any positive integer

8) Suppose you know that every vector in $\mathbb{R}^5$ can be written as a linear combination of the vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$. What can you conclude about n?

(a) $n = 5$
(b) n could be any positive integer
(c) $n \geq 5$
(d) $n \leq 5$

9) Suppose you know that every vector in $\mathbb{R}^5$ can be written uniquely as a linear combination of the vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$. What can you conclude about n?

(a) $n = 5$
(b) n could be any positive integer
(c) $n \leq 5$
(d) $n \geq 5$

10) Suppose you know that every vector in $\mathbb{R}^5$ can be written uniquely as a linear combination of the vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$. What can you conclude about the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$?

(a) It is a basis of $\mathbb{R}^5$.
(b) It does not span and is linearly dependent
(c) It does not span and is linearly independent
(d) It spans but it is linearly dependent