MA437 - 2019 Spring - Assessment 04/11 Mult. Choice - Dr. Clontz

Name: ______________________________

Instructions

Use the provided answer sheet to select the most appropriate response for each multiple choice Computation/Knowledge question, skipping any sections already checked-off as mastered on your progress report.

Chapter 2 Computation

1. Let $f(z) = \frac{z+i}{z^2+1}$. Find $f(2+i)$.
   - **A.** $\frac{1}{2}$
   - B. $3i$
   - C. $4-i$
   - D. $\frac{1}{-1+8i}$
   - E. None of these.
2. Find $\lim_{z \to 1-i} xy + 3ix$ where $z = x + iy$.
   - **A.** $-1+3i$
   - B. $-6i$
   - C. $3-2i$
   - D. 5
   - E. None of these.
3. Find $f'(z)$ given $f(z) = (1+2z)^3$.
   - **A.** $6 + 24z + 24z^2$
   - B. $z^3 - 3z^2 + 3z + 1$
   - C. $2z^2 + i$
   - D. $8iz^3$
   - E. None of these.
4. The function $f(z) = (x^2 - 2x - y^2) + 2yi(x-1)$ is differentiable at $z = -i$. Find $f'(-i)$.
   - **A.** $-2-2i$
   - B. $4-i$
   - C. $6+3i$
   - D. $-2-3i$
   - E. None of these.
5. Which of these describes the differentiable points of of $h(x+iy) = x^2 + y^2 - 2ixy$?
   - **A.** $x = 0$
   - B. $y = 0$
   - C. $x + iy = 0$
   - D. $x = y$
   - E. None of these.

Chapter 3 Computation

6. Simplify $(e^{1+i\pi/2})^3$.

    A. $-ie^3$

    B. $e^3 - i\pi$

    C. $3e^2$

    D. $e\pi + 3ie$

    E. None of these.

7. Which of these describes $\log(e^{2+3i\pi/4})$ where k is any integer?

    A. $2 + i(2k\pi - 5\pi/4)$

    B. $2k + 3i\pi/4$

    C. $2e - 2ki\pi$

    D. $-2 + i(2\pi - 3k\pi/4)$

8. Simplify $\sqrt[4]{16}$.

    A. **± 2 or $\pm 2i$**

    B. $\pm\sqrt{2}(1+i)$ or $\pm\sqrt{2}(i-1)$

    C. $\pm 4(1-i)$ or $\pm 4(i-1)$

    D. $\pm 2(-1+i)$ or $\pm 2(1+i)$

    E. None of these.

9. Describe all complex numbers z such that $e^z = -e$.

    A. $1 + \log(-1)$

    B. $-1 + \mathrm{Log}(1)$

    C. $-e + \log(1)$

    D. $e + \mathrm{Log}(-1)$

    E. None of these.

10. Describe the branch cut of $\mathrm{Log}^\star(z) = |z|e^{i\,\mathrm{Arg}^\star(z)}$ where $\mathrm{Arg}^\star(z)$ is the value of θ such that $z = |z|e^{i\theta}$ and $0 \le \theta < 2\pi$. (Put another way, where is $\mathrm{Log}^\star$ non-analytic?)

    A. $Re(z) \ge 0, Im(z) = 0$

    B. $Re(z) \le 0, Im(z) = 0$

    C. $Im(z) \ge 0, Re(z) = 0$

    D. $Im(z) \le 0, Re(z) = 0$

Chapter 4a Computation

11. Find $\int_0^\pi e^{it}dt$.

    A. $2i$

    B. $-2e$

    C. $-i$

    D. $2e+i$

    E. None of these.

12. Which of these is a parametrization of the line segment connecting $-2i$ to $1+i$ in the complex plane for $0 \leq t \leq 1$?

    A. $z(t) = -2i + t + 3it$

    B. $z(t) = 1 - 2it$

    C. $z(t) = t + 1 + 3it$

    D. $z(t) = 3t - 1 + 2it - i$

    E. None of these.

13. Given the parametrization $z(t) = 3e^{2\pi it}$, what is $|\frac{dz}{dt}|$?

    A. 6π

    B. $3e^{2\pi}$

    C. $2e^3\pi|t|$

    D. $8e|t|$

    E. None of these.

14. Let C be the line segment joining 0 to πi. Find $\int_C e^x(\cos y + i\sin y)dz$ where $z = x + iy$.

    A. -2

    B. 0

    C. $3i$

    D. $\pi + i$

    E. None of these.

15. Find $\int_a^b |z(t)|dt$ where $z(t)$, $a \leq t \leq b$ parameterizes the clockwise boundary of the unit square with corners at $0, i, 1+i, 1$.

    A. 4

    B. 0

    C. 1

    D. i

Chapter 4b Computation

16. Which of these is the smallest upper bound for the value of $\left|\int_C \frac{2z}{z^2+1}dz\right|$ where C is the positively-oriented circle centered at the origin with radius 5? (Remember that it is bounded by the length of C multiplied by an upper bound for $\frac{2z}{z^2+1}$.)

    A. $25\pi/6$

    B. $31\pi/5$

    C. $11\pi/3$

    D. $7\pi/2$

17. Which of these is an antiderivative of $\frac{1}{2z}$ valid whenever z is not a purely negative real number or 0?

    A. $\frac{1}{2}\,\mathrm{Log}(z)$

    B. e^{2z}

    C. $\log(2z)$

    D. $2e^{-z}$

    E. None of these.

18. Let C be the positively-oriented unit circle. Find $\int_C 3e^{z^2}dz$.

    A. 0

    B. $6\pi i$

    C. $-3i$

    D. $2i\pi + 3$

    E. None of these.

19. Let C be the positively-oriented unit circle. Find $\int_C \frac{3}{z}dz$.

    A. 0

    B. $6\pi i$

    C. $-3i$

    D. $2i\pi + 3$

    E. None of these.

20. Let C be the positively-oriented unit circle. Find $\int_C \frac{z+0.5i}{z-0.5i}dz$.

    A. 0

    B. -2π

    C. πi

    D. $1 - i$

    E. None of these.

Chapter 2 Knowledge

21. If $\lim_{z \to z_0} \frac{f(z_0)-f(z)}{z_0-z}$ exists, then f is differentiable at z_0.

    A. True

    B. False

22. If $\lim_{z \to w} f(z) = 0$ and $\lim_{z \to w} g(z) = 0$, then $\lim_{z \to w} \frac{f(z)}{g(z)}$ cannot exist.

    A. True

    B. False

23. If $u_x = -u_y$ at a point, then $f(z) = u(z) + iv(z)$ is differentiable at that point.

    A. True

    B. False

24. If f is differentiable whenever $|z - 2i| < 2$, then f is analytic at $z = i$.

    A. True

    B. False

25. If f and g are entire functions, then their product fg is entire.

    A. True

    B. False

Chapter 3 Knowledge

26. $e^z = e^{z+\pi}$ for all complex numbers z.

    A. True

    B. False

27. $|\operatorname{Log}(z)| > 0$ for all complex z.

    A. True

    B. False

28. The principle value of z^c is based upon the argument of z valued between $-\pi$ and π.

    A. True

    B. False

29. $e^{|z|} \neq 0$ for all complex numbers z.

    A. True

    B. False

30. $i^{1/6}$ has values $\pm z$ for some fixed z.

    A. True

    B. False

Chapter 4a Knowledge

31. $\int_a^b w(t)dt = \int_a^c w(t)dt + \int_c^b w(t)dt$ for all $a < c < b$.

    A. True

    B. False

32. Joining five line segments end-to-end results in a contour.

    A. True

    B. False

33. Let C_0 and C_1 be contours that both start at w_0 and end at w_1. Then $\int_{C_0} f(z)dz = \int_{C_1} f(z)dz$ for all functions f.

    A. True

    B. False

34. $\int_C (z^3 - z + i)dz = 0$ whenever C is a closed contour.

    A. True

    B. False

35. $\int_C \frac{1}{z-i}dz = 0$ whenever C is a circle containing i in its interior.

    A. True

    B. False

Chapter 4b Knowledge

36. $\int_C \frac{f(z)}{z-i}dz = 2\pi i f(i)$ for every closed loop C.

    A. True

    B. False

37. If f is entire, then $\int_C f(z)dz = 0$ for every closed loop C.

    A. True

    B. False

38. If C is a positively oriented simple closed contour containing the positively-oriented unit circle U in its interior, then $\int_C \frac{z^3-z+i}{z}dz = \int_U \frac{z^3-z+i}{z}dz$.

    A. True

    B. False

39. If f is analytic at z, then $f^{(2019)}(z)$ (the 2019th derivative of f at z) always exists.

    A. True

    B. False

40. $\text{Log}(z)$ is analytic for all complex numbers on the domain $Im(z) > 0$, but $\text{Log}\, z$ does not have an antiderivative defined on that same domain.

    A. True

    B. False

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Proofs

Solve at most one of the exercises from each chapter, skipping any chapters already checked-off as mastered on your progress report.

1. **Ch1** Prove that $Im(z) \leq |z|$.
2. **Ch2** Show that $\lim_{z\to i} \frac{1-|z|^2}{z^2+1}$ does not exist.
3. **Ch2** Prove that $\frac{d}{dz}[x^3 + 3xiy - 3xy^2 - iy^3 - 1 + 2i] = 3z^2$ where $z = x + iy$.
4. **Ch3** Find all values of z such that $\text{Log}(z) = 0$.
5. **Ch3** Explain why the principle value of $\sqrt{i}$ is in the first quadrant.
6. **Ch4a** Give a complex integral that can be used to solve the real integral $\int_0^\pi e^{3x}\cos(2x)dx$ without needing integration by parts. Don't actually solve your complex integral, but explain how it can be used to solve the real integral.
7. **Ch4a** Let C be a positively-oriented closed contour and n be an integer. Describe all the possible values of $\int_C \frac{1}{2z^n}dz$ and when they occur.
8. **Ch4b** Let C be the negatively-oriented unit circle. Explain why $\int_C \frac{e^z}{4-z^2}dz = 0$.
9. **Ch4b** Let f be entire and C be the positively-oriented circle of radius 42 centered at the origin. Explain how to simplify $\int_C \frac{3f(z)}{\pi(z-3i)^4}dz$ in terms of f.