Let $V = \{(x,y) : x, y \in \mathbb{R}\}$ have the operations

$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 y_1, x_2 + y_2)$$
$$c \odot (x_1, x_2) = (x_1 + c - 1, c x_2)$$

a) Show that vector addition is associative:

$(\vec{u} \oplus \vec{v}) \oplus \vec{w} = \vec{u} \oplus (\vec{v} \oplus \vec{w})$ for all $\vec{u}, \vec{v}, \vec{w} \in V$.

b) Explain why V nonetheless is <u>NOT</u> a vector space.

a)

$$LHS = (\vec{u} \oplus \vec{v}) \oplus \vec{w} = ((u_1, u_2) \oplus (v_1, v_2)) \oplus \vec{w}$$

$$= (u_1 v_1, u_2 + v_2) \oplus (w_1, w_2)$$

$$= (u_1 v_1 w_1, u_2 + v_2 + w_2)$$

$$RHS = \vec{u} \oplus (\vec{v} \oplus \vec{w}) = \vec{u} \oplus ((v_1, v_2) \oplus (w_1, w_2))$$

$$= (u_1, u_2) \oplus (v_1 + w_1, v_2 w_2)$$

$$= (u_1 v_1 w_1, u_2 + v_2 + w_2)$$

So LHS = RHS, proving the identity true.

b) ~~$1 \odot (2,3) = (2+1-1, 1\cdot 3) = (2,3)$~~

(Gotta try something else, this seems fine.)

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b) $(1+2)\odot(3,4) = 3\odot(3,4)$

$= (3+3-1, 3\cdot 4)$

$= (5,12)$

but $1\odot(3,4) \oplus 2\odot(3,4) = (3+1-1, 1\cdot 4) \oplus (3+2-1, 2\cdot 4)$

$= (3,4) \oplus (4,8)$

$= (3\cdot 4, 4+8)$

$= (12,12) \neq (5,12)$

Since the distrubution property fails, V is <u>not</u> a vector space.

OR

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b) $(2\cdot 2)\odot(0,0) = 4\odot(0,0) = (0+4-1, 4\cdot 0) = (3,0)$

but $2\odot(2\odot(0,0)) = 2\odot(0+2-1, 2\cdot 0) = 2\odot(1,0)$

$= (1+2-1, 2\cdot 0) = (2,0) \neq (3,0)$

Since the associative property fails, V is <u>not</u> a vector space.