

④ Compute $\int_0^{\pi/2} \sec(3\theta) \tan(3\theta) d\theta$.

Let $u = 3\theta$

$\theta = \pi/2 \rightarrow u = 3(\pi/2) = \pi/2$

$\theta = 0 \rightarrow u = 3(0) = 0$

$du = 3d\theta$
 $\frac{1}{3} du = d\theta$

$= \int_{u=0}^{u=\pi/2} \sec(u) \tan(u) \left(\frac{1}{3} du\right)$

$= \left[\frac{1}{3} \sec(u) \right]_0^{\pi/2}$

$= \frac{1}{3} \sec\left(\frac{\pi}{2}\right) - \frac{1}{3} \sec(0)$

$= \frac{1}{3} \sqrt{2} - \frac{1}{3} (1)$

$= \frac{\sqrt{2} - 1}{3}$

$\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)}$

$= \frac{1}{\sqrt{2}/2}$

$= \frac{2}{\sqrt{2}} = \frac{\sqrt{2}\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

(10) Compute $\int_1^2 (6x+3)(x^2+x)^2 dx$.

$$\begin{aligned}\text{Let } u &= x^2+x \\ du &= 2x+1 dx \\ 3du &= 6x+3 dx\end{aligned}$$

$$x=2 \rightarrow u=2^2+2 \\ =4+2=6$$

$$x=1 \rightarrow u=1^2+1 \\ =1+1=2$$

$$= \int_{u=2}^{u=6} 3(u)^2 du$$

$$= [u^3]_2^6$$

$$= 6^3 - 2^3$$

$$= 216 - 8$$

$$= \boxed{208}$$

⑪ Compute $\int_{\ln 3}^{\ln 8} e^z \sqrt{1+e^z} dz$

Let $u = 1 + e^z$
 $du = e^z dz$

$z = \ln 8 \rightarrow u = 1 + e^{\ln 8}$
 $= 9$

$z = \ln 3 \rightarrow u = 1 + e^{\ln 3}$
 $= 4$

$\int_{u=4}^{u=9} \sqrt{u} du$

$= \left[\frac{2}{3} u^{3/2} \right]_4^9$

$= \frac{2}{3} \left[9^{3/2} - 4^{3/2} \right]$

$= \frac{2}{3} \left[3^3 - 2^3 \right]$

$= \frac{2}{3} \left[27 - 8 \right]$

$= \frac{2}{3} \left[19 \right] = \boxed{\frac{38}{3}}$

(12)

Compute $\int_e^{e^2} \frac{1}{x \ln x} dx$.

$$\text{Let } u = \ln x \\ du = \frac{1}{x} dx$$

$$x = e^2 \rightarrow u = \ln e^2 \\ = 2$$

$$x = e \rightarrow u = \ln e \\ = 1$$

$$\int_{u=1}^{u=2} \frac{1}{u} du$$

$$= [\ln|u|]_1^2$$

$$= \ln 2 - \cancel{\ln 1}$$

$$= \boxed{\ln 2}$$

(13) Find $\int \cot \theta \, d\theta$.

$$= \int \frac{\cos \theta}{\sin \theta} \, d\theta$$

$$\text{Let } u = \sin \theta \\ du = \cos \theta \, d\theta$$

$$= \int \frac{1}{u} \, du$$

$$= \ln |u| + C$$

$$= \boxed{\ln |\sin \theta| + C}$$

OR

$$= -\ln |(\sin \theta)^{-1}| + C$$

$$= -\ln \left| \frac{1}{\sin \theta} \right| + C$$

$$= \boxed{-\ln |\csc \theta| + C}$$

(14) Find $\int \sec x \, dx$.

$$= \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} \, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

Let $u = \sec x + \tan x$
 $du = \sec x \tan x + \sec^2 x \, dx$

$$= \int \frac{1}{u} \, du$$

$$= \ln|u| + C$$

$$= \boxed{\ln|\sec x + \tan x| + C}$$

①5 Find $\int 3t^5(t^3+3)^2 dt$

$$= \int t^3(t^3+3)^2 3t^2 dt$$

Let $u = t^3 + 3 \rightarrow t^3 = u - 3$
 $du = 3t^2 dt$

$$= \int (u-3)(u)^2 du$$

$$= \int u^3 - 3u^2 du$$

$$= \frac{1}{4}u^4 - u^3 + C$$

$$= \frac{1}{4}(t^3+3)^4 - (t^3+3)^3 + C$$

(16) Evaluate $\int_0^1 x^2 e^{2x^3} dx$.

$$\begin{aligned}\text{Let } u &= 2x^3 \\ du &= 6x^2 dx \\ \frac{1}{6} du &= x^2 dx\end{aligned}$$

$$x=1 \rightarrow u = 2(1)^3 = 2$$

$$x=0 \rightarrow u = 2(0)^3 = 0$$

$$= \int_{u=0}^{u=2} \frac{1}{6} e^u du$$

$$= \left[\frac{1}{6} e^u \right]_0^2$$

$$= \frac{1}{6} e^2 - \frac{1}{6} e^0$$

$$= \boxed{\frac{1}{6} e^2 - \frac{1}{6}}$$