

① Find  $\int \sin^4 x \cos^3 x dx$

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$$= \int \sin^4 x \cos^2 x \cos x dx$$

$$= \int \sin^4 x (1 - \sin^2 x) \cos x dx$$

$$\text{Let } u = \sin x \\ du = \cos x dx$$

$$= \int u^4 (1 - u^2) du$$

$$= \int u^4 - u^6 du$$

$$= \frac{1}{5} u^5 - \frac{1}{7} u^7 + C$$

$$= \boxed{\frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C}$$

$$\textcircled{2} \text{ Find } \int \sin^5 \theta \cos^2 \theta \, d\theta.$$

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$$= \int \sin^4 \theta \cos^2 \theta \sin \theta \, d\theta$$

$$= \int (1 - \cos^2 \theta)^2 \cos^2 \theta \sin \theta \, d\theta$$

$$\text{Let } u = \cos \theta$$

$$du = -\sin \theta \, d\theta$$

$$-du = \sin \theta \, d\theta$$

$$= \int (1 - u^2)^2 u^2 (-du)$$

$$= \int (1 - 2u^2 + u^4) u^2 (-du)$$

$$= \int -u^2 + 2u^4 - u^6 \, du$$

$$= -\frac{1}{3}u^3 + \frac{2}{5}u^5 - \frac{1}{7}u^7 + C$$

$$= \boxed{-\frac{1}{3} \cos^3 \theta + \frac{2}{5} \cos^5 \theta - \frac{1}{7} \cos^7 \theta + C}$$

③ Find  $\int \sin^2 x dx$ .

$$= \int \frac{1}{2} - \frac{1}{2} \cos 2x dx$$

$$= \left[ \frac{1}{2} x - \frac{1}{4} \sin 2x + C \right]$$

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④ Find  $\int \cos^4 y dy$ .

$$= \int \left( \frac{1}{2} + \frac{1}{2} \cos 2y \right)^2 dy$$

$$= \int \frac{1}{4} + \frac{1}{2} \cos 2y + \frac{1}{4} \cos^2 2y dy$$

$$= \int \frac{1}{4} + \frac{1}{2} \cos 2y + \frac{1}{4} \left( \frac{1}{2} + \frac{1}{2} \cos 4y \right) dy$$

$$= \int \frac{3}{8} + \frac{1}{2} \cos 2y + \frac{1}{8} \cos 4y dy$$

$$= \left[ \frac{3}{8} y + \frac{1}{4} \sin 2y + \frac{1}{32} \sin 4y + C \right]$$

⑤ Find  $\int \tan^2 t \sec^4 t dt$ .

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$$= \int \tan^2 t \sec^2 t \sec^2 t dt$$

$$= \int \tan^2 t (\tan^2 t + 1) \sec^2 t dt$$

$$\text{Let } u = \tan t \\ du = \sec^2 t dt$$

$$= \int u^2 (u^2 + 1) du$$

$$= \int u^4 + u^2 du$$

$$= \frac{1}{5} u^5 + \frac{1}{3} u^3 + C$$

$$= \boxed{\frac{1}{5} \tan^5 t + \frac{1}{3} \tan^3 t + C}$$