6 Find $\int \tan z \sec^5 z \, dz$.

$$= \int \sec^4 z \quad \sec z \tan z \, dz$$

Let $u = \sec z$

$du = \sec z \tan z \, dz$

$$= \int u^4 \, du$$

$$= \frac{1}{5} u^5 + C$$

$$= \boxed{\frac{1}{5} \sec^5 z + C}$$

② Find $\int \tan^3 x\,dx$.

$$= \int \tan^2 x \tan x \, dx$$

$$= \int (\sec^2 x - 1) \tan x \, dx$$

$$= \int (\tan x \sec^2 x - \tan x) dx$$

$$= \int \tan x \sec^2 x \, dx - \int \tan x \, dx$$

$\left(\int \frac{\sin x}{\cos x} dx = \ln|\sec x| + C \right.$

Let $u = \tan x$
$du = \sec^2 x \, dx$

$$= \int u \, du - \int \tan x \, dx$$

$$= \frac{1}{2} u^2 - \ln|\sec x| + C$$

$$= \boxed{\frac{1}{2} \tan^2 x - \ln|\sec x| + C}$$

(8) Find $\int \frac{\sin^3 r}{\cos^7 r}\,dr$.

$$= \int \frac{\sin^3 r}{\cos^3 r}\,\frac{1}{\cos^4 r}\,dr$$

$$= \int \tan^3 r \sec^4 r\,dr$$

$$= \int \tan^3 r \sec^2 r \sec^2 r\,dr$$

$$\left(\sec^2 r = \tan^2 r + 1\right)$$

$$= \int \tan^3 r\,(\tan^2 r + 1)\sec^2 r\,dr$$

let $u = \tan r$

$du = \sec^2 r\,dr$

$$= \int u^3(u^2+1)\,du$$

$$= \int (u^5 + u^3)\,du$$

$$= \frac{1}{6}u^6 + \frac{1}{4}u^4 + C$$

$$= \boxed{\frac{1}{6}\tan^6 r + \frac{1}{4}\tan^4 r + C}$$

(4) Find $\int_0^{\pi} \sqrt{2-2\cos y}\, dy$.

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\sin^2\left(\frac{y}{2}\right) = \frac{1}{2} - \frac{1}{2}\cos\left(\cancel{2}\frac{y}{\cancel{2}}\right)$$

$$4\sin^2\left(\frac{y}{2}\right) = 2 - 2\cos(y)$$

$$= \int_0^{\pi} \sqrt{4\sin^2\left(\frac{y}{2}\right)}\, dy$$

$$= \int_0^{\pi} 2\left|\sin\left(\frac{y}{2}\right)\right| dy$$

($\sqrt{u^2} = |u|$ since it must be positive.)

$$= \int_0^{\pi} 2\sin\left(\frac{y}{2}\right) dy$$

($\sin\left(\frac{y}{2}\right)$ is always positive between 0 and 2π.)

Let $u = y/2$ $\qquad$ $y = \pi \rightarrow u = \pi/2$

$du = \frac{1}{2} dy$ $\qquad$ $y = 0 \rightarrow u = 0$

$2du = dy$

$$= \int_0^{\pi/2} 2\sin(u)\, 2du$$

$$= \Big[-4\cos(u)\Big]_0^{\pi/2} = -4\cos(\pi/2) + 4\cos(0)$$

$$= +4 + 4 = \boxed{8}$$