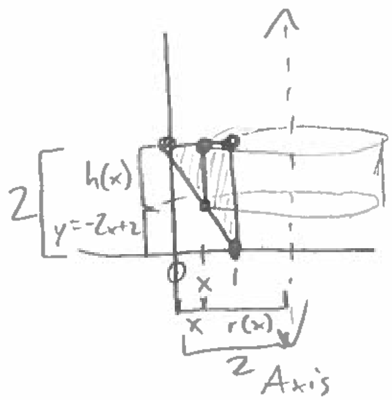
① Find the solid of revolution obtained by rotating the triangle with vertices $(0,2)$, $(1,0)$, $(1,2)$ around the axis $x=2$. (Cylindrical Shell Method)



$h(x)+(-2x+2)=2$

$h(x)=2x$

$r(x)+x=2$

$r(x)=2-x$

$$V=2\pi\int_0^1 (2-x)(2x)\,dx$$

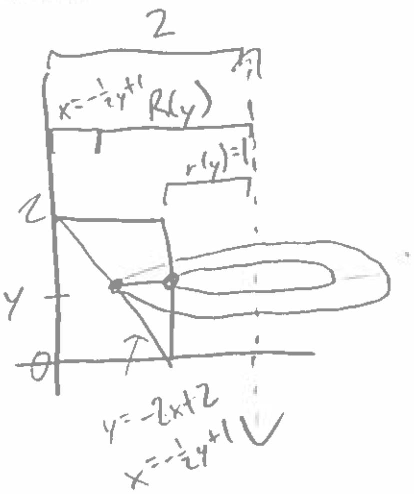
$$=2\pi\int_0^1 (4x-2x^2)\,dx$$

$$=2\pi\left[2x^2-\frac{2}{3}x^3\right]_0^1$$

$$=2\pi\left[(2-2/3)-\cancel{(0-0)}\right]$$

$$=2\pi(4/3)=\boxed{8/3\,\pi}$$

1 using Washer Method instead:



$$R(y)+\left(-\tfrac{1}{2}y+1\right)=2$$

$$R(y)=\tfrac{1}{2}y+1$$

$$V=\pi\int_0^2\left(\left(\tfrac{1}{2}y+1\right)^2-(1)^2\right)dy$$

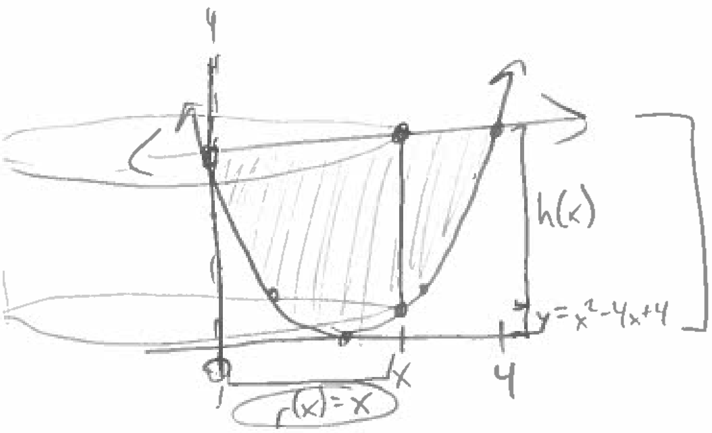
$$=\pi\int_0^2\left(\tfrac{1}{4}y^2+y+\cancel{1-1}\right)dy$$

$$=\pi\left[\tfrac{1}{12}y^3+\tfrac{1}{2}y^2\right]_0^2$$

$$=\pi\left[\left(\cancel{\tfrac{8}{12}}^{2/3}+2\right)-\cancel{(0+0)}\right]$$

$$=\boxed{\tfrac{8}{3}\pi}$$

② Find the volume of the solid of revolution obtained by rotating the region bounded by $y=4$, $y=x^2-4x+4$ around the y-axis. ((Cylindrical Shell Method)



x	$y=x^2-4x+4$
0	4
1	1
2	0
3	1
4	4

$h(x)+(x^2-4x+4)=4$

$h(x)=4x-x^2$

$$V=2\pi\int_0^4 (x)(4x-x^2)\,dx$$

$$=2\pi\int_0^4 (4x^2-x^3)\,dx$$

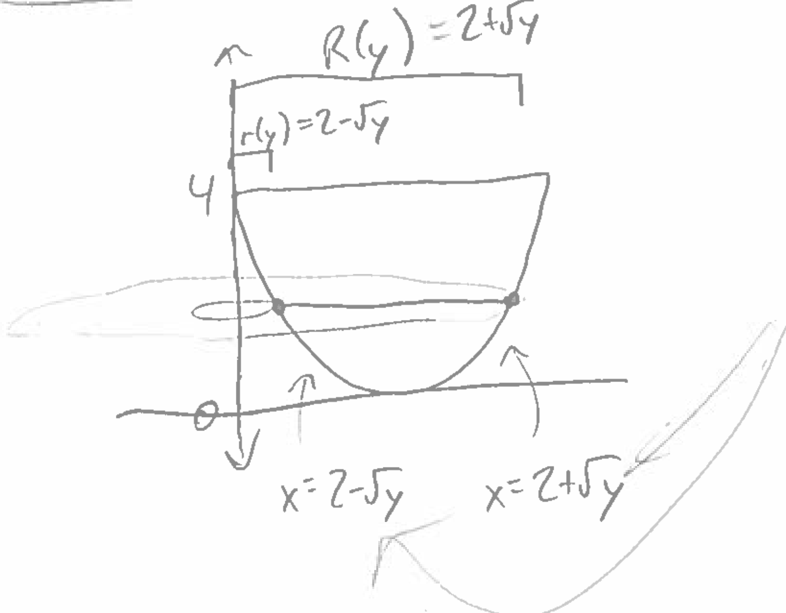
$$=2\pi\left[\tfrac{4}{3}x^3-\tfrac{1}{4}x^4\right]_0^4$$

$$=2\pi\left[\left(\tfrac{1}{3}4^4-4^3\right)-\cancel{(0-0)}\right]$$

$$=2\pi 4^3\left[\tfrac{4}{3}-1\right]$$

$$=\frac{2\pi 4^3}{3}=\boxed{\frac{128\pi}{3}}$$

② using Washer Method instead:



$$y = x^2 - 4x + 4$$
$$y = (x-2)^2$$
$$\pm\sqrt{y} = x - 2$$
$$x = 2 \pm \sqrt{y}$$

$$V = \pi \int_0^4 \left((2+\sqrt{y})^2 - (2-\sqrt{y})^2 \right) dy$$

$$= \pi \int_0^4 \left((\cancel{4} + 4\sqrt{y} + \cancel{y}) - (\cancel{4} - 4\sqrt{y} + \cancel{y}) \right) dy$$

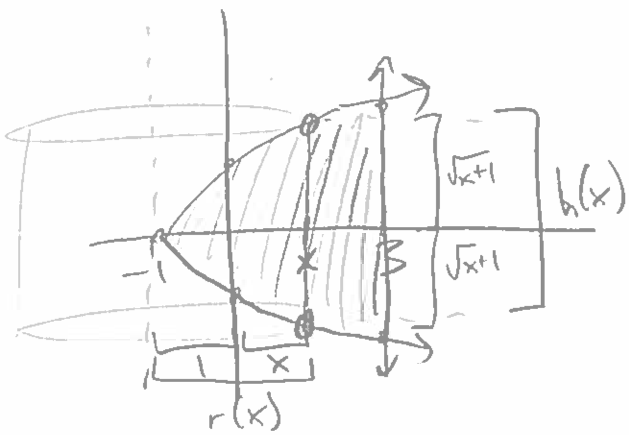
$$= 8\pi \int_0^4 y^{1/2}\, dy$$

$$= 8\pi \left[\frac{2}{3} y^{3/2} \right]_0^4$$

$$= \frac{16}{3}\pi \left[(4)^{3/2} - \cancel{(0)^{3/2}} \right)$$

$$= \frac{16}{3}\pi (8) = \boxed{\frac{128\pi}{3}}$$

(3) Find the volume of the solid of revolution obtained by rotating the region bounded by $x = y^2 - 1$, $x = 3$ around the axis $x = -1$. ((Cyl. Shell Meth.)



$$y^2 = x + 1$$
$$y = \pm\sqrt{x+1}$$

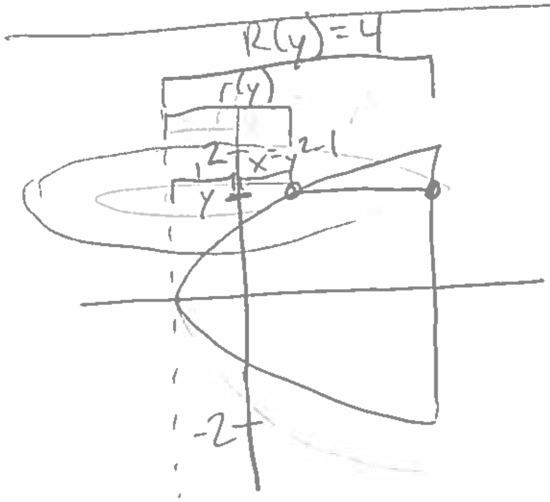
$$h(x) = \sqrt{x+1} + \sqrt{x+1}$$
$$h(x) = 2\sqrt{x+1}$$
$$r(x) = x + 1$$

$$V = 2\pi \int_{-1}^{3} (x+1)(2\sqrt{x+1})\,dx$$

Let $u = x+1$ $\quad x = 3 \to u = 4$
$du = dx$ $\quad x = -1 \to u = 0$

$$= 2\pi \int_0^4 u(2u^{1/2})\,dx$$
$$= 4\pi \int_0^4 u^{3/2}\,dx$$
$$= 4\pi \frac{2}{5}\left[u^{5/2}\right]_0^4$$
$$= \frac{8\pi}{5}\left[4^{5/2} - \cancel{0^{5/2}}\right]$$
$$= \frac{8\pi}{5}(32)$$
$$= \boxed{\frac{256\pi}{5}}$$

③ using Washer Method instead.



$$r(y) = 1 + (y^2 - 1)$$
$$r(y) = y^2$$

$$V = \pi \int_{-2}^{2} \left((4)^2 - (y^2)^2 \right) dy$$
$$= \pi \int_{-2}^{2} (16 - y^4)\, dy$$
$$= \pi \left[16y - \frac{1}{5} y^5 \right]_{-2}^{2}$$
$$= \pi \left[\left(32 - \frac{32}{5}\right) - \left(-32 + \frac{32}{5}\right) \right]$$
$$= \pi \left[4\frac{32}{5} + 4\frac{32}{5} \right]$$
$$= 8\frac{32}{5}\pi = \boxed{\frac{256}{5}\pi}$$