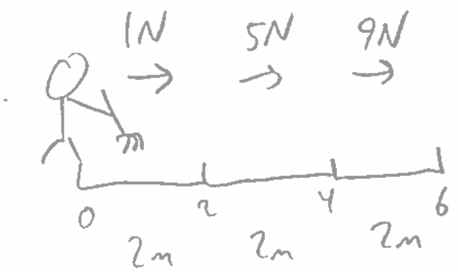
(1) Estimate the work done in pushing a plow 6 m through increasingly packed dirt; this movement requires 1N of force initially, then 5N of force after 2 m, and then 9N of force after 4m.



$$W \approx 1(2)+5(2)+9(2)$$
$$\approx 2+10+18 = \boxed{30\ J}$$

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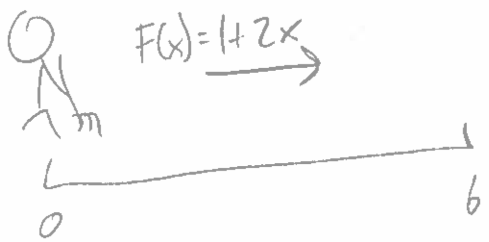
(2) Compute the work done in pushing a plow 6m through increasingly packed dirt; this movement requires $F(x) = 1+2x$ newtons of force after x meters.



$$W = \int_0^6 (1+2x)\,dx$$
$$= \left[x+x^2\right]_0^6$$
$$= (6+36)-\cancel{(0+0)}$$
$$= \boxed{42\ J}$$

(3) Find the work done in lifting a leaky bucket from the ground to a height of 4 feet, assuming it weighs 25-x pounds at x feet above the ground.

$$F(x) = 25 - x$$

$$W = \int_0^4 (25-x)\,dx = \left[25x - \frac{1}{2}x^2\right]_0^4$$

$$= \left(100 - \frac{1}{2}(16)\right) - \cancel{(0-0)}$$

$$= \boxed{92 \text{ ft-lbs}}$$

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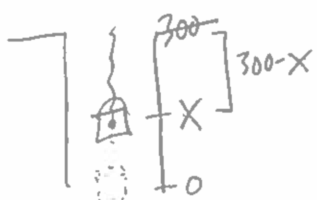
(4) A cable weighing 4 lbs per ft holds a 500 lb bucket of coal at the bottom of a 300 ft mine shaft. Show that the work done in lifting the bucket and cable is 330,000 ft-lbs.

$$F(x) = \underbrace{500}_{\text{bucket}} + 4\underbrace{(300-x)}_{\text{length of cable}} = 1700 - 4x$$

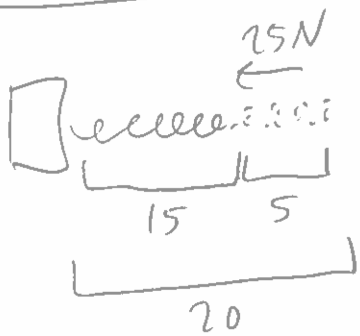


$$W = \int_0^{300} (1700-4x)\,dx = \left[1700x - 2x^2\right]_0^{300}$$

$$= (510000 - 180000) - \cancel{(0-0)}$$

$$= \boxed{330000}$$

⑤ Show that if a spring has natural length 20 cm, and it requires 25 N of force to hold the spring at 15 cm, then the work required to stretch the spring from its natural length to 26 cm is 90 N-cm.

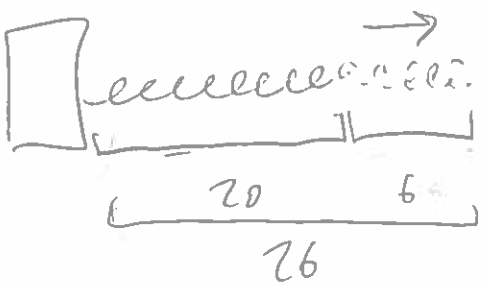


$F(x) = kx$ ← Hooke's Law

$$F(5) = k(5) = 25$$

$$5k = 25$$

$$k = 5$$

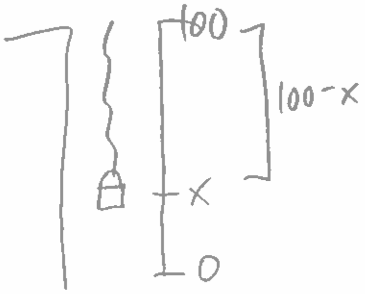


$$W = \int_0^6 F(x)\,dx = \int_0^6 5x\,dx$$

$$= \left[\tfrac{5}{2}x^2\right]_0^6 = \tfrac{5}{2}(36) - \cancel{\tfrac{5}{2}(0)}$$

$$= \boxed{90}$$

(6) A uniformly weighted 100-ft rope weighs 50 lbs. Suppose it is fully extended into a well, tied to a leaky bucket of water. The bucket weighs 10 lbs and initially holds 30 lbs of water, but loses 1 lb of water every 2 ft. Show that the work done in lifting the rope and bucket is 4400 ft-lbs.



Bucket Weight = 10

Work lifting bucket $= \int_0^{100} 10\,dx = 1000$

Rope Weight $= \frac{50}{100}(100-x)$

$= 50 - \frac{1}{2}x$

Work lifting rope $= \int_0^{100} \left(50 - \frac{1}{2}x\right)dx$

$= 2500$

Water Weight $= 30 - \frac{1}{2}x$ ← Note all water gone at 60 ft

Work lifting water $= \int_0^{60} \left(30 - \frac{1}{2}x\right)dx$

$= 900$

Total Work $= 1000 + 2500 + 900 = \boxed{4400}$