① Find the line tangent to the curve parameterized by $x=t^2$, $y=t^3$ at the point where $t=-2$.

$\frac{dx}{dt}=2t$ $\quad$ $\frac{dy}{dt}=3t^2$

$=-4$ $\quad$ $=12$

$x=t^2$
$=4$

$y=t^3$
$=-8$

$dy/dt = dy/dx \; dx/dt$

$12=-4\,dy/dx$

$-3=dy/dx=\text{slope}$

$(4,-8)=\text{point}$

$y-y_0=m(x-x_0)$

$y+8=-3(x-4)$

$y+8=-3x+12$

$\boxed{y=-3x+4}$

② Show that the curve parameterized by $x = 3\sin t$, $y = 3\cos t$ at $(3/2, 3\sqrt{3}/2)$ has the EQ $y = 2\sqrt{3} - 1/\sqrt{3}\,x$

point

$$x = 3\sin t = 3/2$$
$$\sin t = 1/2$$
$$t = \pi/6 \text{ OR } 5\pi/6$$

$$y = 3\cos t = 3\sqrt{3}/2$$
$$\cos t = \sqrt{3}/2$$
$$t = \pi/6 \text{ or } -\pi/6$$

$$t = \pi/6$$

$$dx/dt = 3\cos t = 3\left(\tfrac{\sqrt{3}}{2}\right)$$

$$dy/dt = -3\sin t = -3\left(\tfrac{1}{2}\right)$$

$$dy/dt = dy/dx \; dx/dt$$
$$-3/2 = dy/dx \; 3\sqrt{3}/2$$
$$-\frac{\cancel{3}}{\cancel{2}}\frac{\cancel{2}}{\cancel{3}\sqrt{3}} = dy/dx$$
$$-1/\sqrt{3} = dy/dx = \text{slope}$$

$$y - y_0 = m(x - x_0)$$
$$y - 3\sqrt{3}/2 = -\frac{1}{\sqrt{3}}\left(x - \frac{3}{2}\right)$$
$$y - 3\sqrt{3}/2 = -\frac{1}{\sqrt{3}}x + \frac{\sqrt{3}\,\cancel{3}}{\cancel{\sqrt{3}}\,2}$$
$$y = -\frac{1}{\sqrt{3}}x + 4\sqrt{3}/2$$
$$\boxed{y = 2\sqrt{3} - 1/\sqrt{3}\,x}$$

(3) Find the point on $x = 2t^2+1$, $y = t^4 - 4t$ which has a horizontal tangent line.

$$dy/dt = dy/dx \; dx/dt$$
$$dy/dt = 0 \; dx/dt$$
$$dy/dt = 0$$
$$4t^3 - 4 = 0$$
$$4t^3 = 4$$
$$t^3 = 1$$
$$t = 1$$

$$x = 2(1)^2+1$$
$$= 3$$

$$y = (1)^4 - 4$$
$$= -3$$

$$\boxed{(3,-3)}$$

(4) Use the arclength formula to find the length of the line segment joining $(-2,6)$ and $(3,-6)$.

Distance formula:

$$L=\sqrt{(3-{-2})^2+(-6-6)^2}=\sqrt{25+144}=13$$

Arclength formula:

$$L=\int_a^b \sqrt{(dx/dt)^2+(dy/dt)^2}\,dt$$

$$=\int_0^1 \sqrt{5^2+(-12)^2}\,dt$$

$$=\int_0^1 \sqrt{25+144}\,dt$$

$$=\int_0^1 \sqrt{169}\,dt$$

$$=\Big[13t\Big]_0^1$$

$$=13-0$$

$$=\boxed{13}$$

Line Seg. Para. EQs:

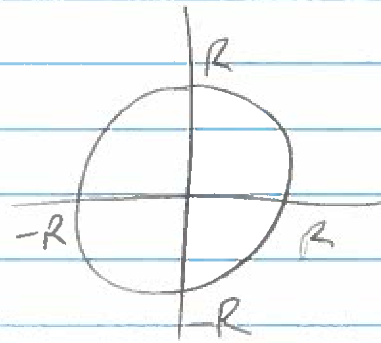
$0\le t\le 1$

$$x=x_0+(x_1-x_0)t$$
$$=-2+(3-{-2})t$$
$$=-2+5t$$

$$y=y_0+(y_1-y_0)t$$
$$=6+(-6-6)t$$
$$=6-12t$$

$$dx/dt=5$$
$$dy/dt=-12$$

(5) Use the arclength formula to prove that the circumference of a circle of radius R is $2\pi R$.



$$x^2+y^2=R^2$$
$$=R^2\cos^2 t+R^2\sin^2 t$$

$x=R\cos t$
$y=R\sin t$ $\quad 0\le t\le 2\pi$

$dx/dt=-R\sin t$
$dy/dt=R\cos t$

$$L=\int_0^{2\pi}\sqrt{(-R\sin t)^2+(R\cos t)^2}\,dt$$
$$=\int_0^{2\pi}\sqrt{R^2\sin^2 t+R^2\cos^2 t}\,dt$$
$$=\int_0^{2\pi}\sqrt{R^2}\,\cancel{\sqrt{\sin^2 t+\cos^2 t}}\,dt$$
$$=\left[Rt\right]_0^{2\pi}=\boxed{2\pi R}$$

(6) Show that the arclength of the curve parameterized by $x = \cos 2t$, $y = 2t + \sin 2t$, $0 \le t \le \pi/2$ is 4.

$$L = \int_0^{\pi/2} \sqrt{(-2\sin 2t)^2 + (2 + 2\cos 2t)^2}\, dt$$

$$= \int_0^{\pi/2} \sqrt{4\sin^2 2t + 4 + 8\cos 2t + 4\cos^2 2t}\, dt$$

$$= \int_0^{\pi/2} \sqrt{4(\cancel{\sin^2 2t + \cos^2 2t}) + 4 + 8\cos 2t}\, dt$$

$$= \int_0^{\pi/2} \sqrt{8 + 8\cos 2t}\, dt$$

$$= \int_0^{\pi/2} \sqrt{16\cos^2 t}\, dt$$

$$= \int_0^{\pi/2} 4|\cos t|\, dt = \int_0^{\pi/2} 4\cos t\, dt$$

$$= \left[4\sin t\right]_0^{\pi/2}$$

$$= 4(1) - \cancel{4(0)} = \boxed{4}$$

(2) Find the area of the surface obtained by rotating the curve parameterized by $x = \cos t$, $y = 2 + \sin t$, $0 \le t \le \pi/2$ around the x-axis.

$$A = 2\pi \int_a^b y \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt$$

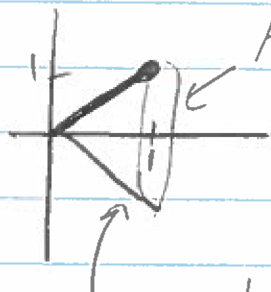
$$= 2\pi \int_0^{\pi/2} (2 + \sin t) \cancel{\sqrt{(-\sin t)^2 + (\cos t)^2}}\, dt$$

$$= 2\pi \Big[2t - \cos t \Big]_0^{\pi/2}$$

$$= 2\pi \big[(\pi - 0) - (0 - 1) \big]$$

$$= \boxed{2\pi^2 + 2\pi}$$

⑧ Use the para. EQs $x=t$, $y=t$, $0 \le t \le 1$ to show that the surface area of a cone of height 1 and radius 1 is $\pi(\sqrt{2}+1)$.



$$A = 2\pi \int_a^b y \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt$$

$$= 2\pi \int_0^1 t \sqrt{1 + 1}\, dt$$

$$= 2\sqrt{2}\pi \int_0^1 t\, dt$$

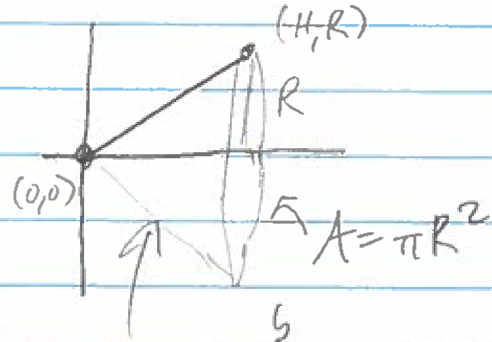
$$= \cancel{2}\sqrt{2}\pi \left[\cancel{\tfrac{1}{2}} t^2\right]_0^1$$

$$= \sqrt{2}\pi$$

Total Area $= \sqrt{2}\pi + \pi$

$$= \boxed{\pi(\sqrt{2}+1)}$$

(4) Show that the surface area of the cone of height H and radius R is $\pi R\left(\sqrt{H^2+R^2}+R\right)$.



$x = Ht$ $\quad 0 \le t \le 1$

$y = Rt$

$$A = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$= 2\pi \int_0^1 Rt\sqrt{H^2+R^2}\,dt$$

$$= 2\pi R\sqrt{H^2+R^2}\int_0^1 t\,dt$$

$$= \cancel{2}\pi R\sqrt{H^2+R^2}\left[\frac{1}{\cancel{2}}t^2\right]_0^1$$

$$= \pi R\sqrt{H^2+R^2}$$

Total Area $= \pi R\sqrt{H^2+R^2} + \pi R^2$

$$= \boxed{\pi R\left(\sqrt{H^2+R^2}+R\right)}$$

(10) Find a point on $x = e^{3t} + 5$, $y = e^{2t} - 2t + 1$ which has a horizontal tangent line.

$dy/dt = dy/dx \; dx/dt$

$dy/dt = 0 \; dx/dt$

$dy/dt = 0$

$2e^{2t} - 2 = 0$

$2e^{2t} = 2$

$e^{2t} = 1$

$2t = 0$

$t = 0$

$x = e^0 + 5$

$= 1 + 5$

$= 6$

$y = e^0 - 2(0) + 1$

$= 1 - 0 + 1$

$= 2$

$\boxed{(6, 2)}$

(11) Find an integral which gives the length of the curve $y = x^2 - 3x + 4$ between $(1,2)$ and $(3,4)$.

$x = t$ $\qquad dx/dt = 1$

$y = t^2 - 3t + 4$ $\qquad dy/dt = 2t - 3$

$1 \le t \le 3$

$$L = \int_1^3 \sqrt{(1)^2 + (2t-3)^2}\, dt$$

$$= \int_1^3 \sqrt{1 + 4t^2 - 12t + 9}\, dt$$

$$= \boxed{\int_1^3 \sqrt{4t^2 - 12t + 10}\, dt}$$