① Find $\lim_{n\to\infty} \frac{n-4n^2}{2n^2+7}$.

$$= \lim_{n\to\infty} \frac{\cancel{n^2}\left(\frac{n}{n^2} - \frac{4n^2}{n^2}\right)}{\cancel{n^2}\left(\frac{2n^2}{n^2} + \frac{7}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n} - 4}{2 + \frac{7}{n^2}} = \frac{0-4}{2+0} = -\frac{4}{2} = \boxed{-2}$$

② Prove that $\frac{\ln n}{n} \to 0$.

$$\lim_{n\to\infty} \frac{\ln n}{n} = \lim_{x\to\infty} \frac{\ln x}{x} \cancel{\frac{\infty}{\infty}} \overset{LH}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}[\ln x]}{\frac{d}{dx}[x]} = \lim_{x\to\infty} \frac{1/x}{1} = \frac{0}{1} = \boxed{0}$$

③ Find $\lim_{n\to\infty} \frac{\cos n}{n \ln n}$.

$$\frac{-1}{n\ln n} \leq \frac{\cos n}{n \ln n} \leq \frac{1}{n \ln n}$$

$$0 = \lim_{n\to\infty} \frac{-1}{n\ln n} \leq \lim_{n\to\infty} \frac{\cos n}{n\ln n} \leq \lim_{n\to\infty} \frac{1}{n\ln n} = 0$$

So $\lim_{n\to\infty} \frac{\cos n}{n \ln n} = \boxed{0}$

④ Find $\lim_{n\to\infty} \frac{\sin n + 3n^2}{n^2+1}$.

$$\frac{-1}{n^2+1} \le \frac{\sin n}{n^2+1} \le \frac{1}{n^2+1}$$

$$0 = \lim_{n\to\infty} \frac{-1}{n^2+1} \le \lim_{n\to\infty} \frac{\sin n}{n^2+1} \le \lim_{n\to\infty} \frac{1}{n^2+1} = 0$$

So $\lim_{n\to\infty} \frac{\sin n}{n^2+1} = 0$.

$$\lim_{n\to\infty} \frac{3n^2}{n^2+1} = \lim_{n\to\infty} \frac{\cancel{n^2}}{\cancel{n^2}} \frac{3}{1+\frac{1}{n^2}} = \frac{3}{1+0} = 3$$

$$\lim_{n\to\infty} \frac{\sin n + 3n^2}{n^2+1} = \lim_{n\to\infty} \frac{\sin n}{n^2+1} + \lim_{n\to\infty} \frac{3n^2}{n^2+1}$$

$$= 0 + 3$$

$$= \boxed{3}$$

5. Find $\lim_{n\to\infty} \frac{\ln(n^n)}{n^2}$.

$$= \lim_{n\to\infty} \frac{\cancel{n} \ln(n)}{n^{\cancel{2}}} = \lim_{n\to\infty} \frac{\ln(n)}{n} = \boxed{0}$$

7. Find $\lim_{n\to\infty} \left(\frac{1}{\pi}\right)^{3n}$.

$$= \left(\lim_{n\to\infty} \left(\frac{1}{\pi}\right)^n\right)^3 = (0)^3 = \boxed{0}$$

6. Find $\lim_{n\to\infty} (5n^3)^{2/n}$.

$$= \lim_{n\to\infty} 5^{2/n}(n^3)^{2/n} = \lim_{n\to\infty} \left(25^{\frac{1}{n}}\right)\left(n^{\frac{1}{n}}\right)^6$$

$$= \left(\lim_{n\to\infty} \sqrt[n]{25}\right)\left(\lim_{n\to\infty} \sqrt[n]{n}\right)^6 = (1)(1)^6 = \boxed{1}$$

8. Find $\lim_{n\to\infty} \left(\frac{1}{2} + \frac{1}{n}\right)^n$.

$$= \lim_{n\to\infty} \left(\left(\frac{1}{2}\right)\left(1 + \frac{2}{n}\right)\right)^n = \left(\lim_{n\to\infty} \left(\frac{1}{2}\right)^n\right)\left(\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^n\right)$$

$$= (0)(e^2) = \boxed{0}$$

9) Find $\lim_{n\to\infty} \dfrac{\frac{(n+2)!}{2^n}}{\frac{3n^2 n!}{2^{n+1}}}$.

$$= \lim_{n\to\infty} \frac{(n+2)!}{2^n} \frac{2^{n+1}}{3n^2 n!}$$

$$\left((n+2)! = \underbrace{(1)(2)(3)\cdots(n)}_{= n!}(n+1)(n+2) \right)$$

$$= \lim_{n\to\infty} \frac{\cancel{n!}(n+1)(n+2)}{\cancel{2^n}} \frac{\cancel{2^n}\, 2^1}{3n^2 \cancel{n!}}$$

$$= \frac{2}{3} \lim_{n\to\infty} \frac{(n+1)(n+2)}{(n)(n)}$$

$$= \frac{2}{3} \lim_{x\to\infty} \left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)$$

$$= \frac{2}{3}(1+0)(1+0) = \boxed{\frac{2}{3}}$$

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10) Does $\left\langle \frac{2+n^2}{n^2-1} \right\rangle_{n=2}^{\infty}$ appear bounded? Monotonic? To converge?

$$= \left\langle \frac{6}{3}, \frac{11}{8}, \frac{18}{15}, \frac{27}{24}, \frac{38}{35}, \ldots \right\rangle$$

$$\approx \langle 2.00, 1.38, 1.20, 1.13, 1.09, \ldots \rangle$$

$\boxed{\text{Monotonic: Yes}}$ (never increases)

$\boxed{\text{Bounded: Yes}}$ (between 1 and 2)

$\boxed{\text{Converges: Yes}}$ (By monotonic sequence Thm)

(11) Does $\langle (-3)^n \rangle_{n=0}^{\infty}$ appear bounded? Monotonic? To converge?

$= \langle 1, -3, 9, -27, 81, -243, \ldots \rangle$

$\boxed{\text{Monotonic: No}}$ (alternates between + and −)

$\boxed{\text{Bounded: No}}$ $(1, 9, 81, \cdots \to \infty)$

$\boxed{\text{Converges: No}}$ (Doesn't approach any real number)

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(12) Does $\langle (-\frac{1}{2})^n \rangle_{n=1}^{\infty}$ appear bounded? Monotonic? To converge?

$= \langle -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{72}, \frac{1}{64}, \ldots \rangle$

$\boxed{\text{Bounded: Yes}}$ (between −1 and 1)

$\boxed{\text{Monotonic: No}}$ (alternates between + and −)

$\boxed{\text{Converges: Yes}}$ (Monotonic Sequence Thm doesn't apply, but appears to converge to 0.)

(13) Prove $\lim\limits_{n \to \infty} \left(1+\frac{x}{n}\right)^n = e^x$.

Let $L = \lim\limits_{t \to \infty} \left(1+\frac{x}{t}\right)^t$

$$\ln L = \ln\left(\lim_{t \to \infty} \left(1+\frac{x}{t}\right)^t\right)$$

$$= \lim_{t \to \infty} \ln\left(\left(1+\frac{x}{t}\right)^t\right)$$

$$= \lim_{t \to \infty} t \ln\left(1+\frac{x}{t}\right)$$

$$= \lim_{t \to \infty} \frac{\ln\left(1+\frac{x}{t}\right)}{\frac{1}{t}} \quad = \frac{0}{0}$$

LH form

$$= \lim_{t \to \infty} \frac{d/dt\left[\ln\left(1+\frac{x}{t}\right)\right]}{d/dt\left[\frac{1}{t}\right]}$$

$$= \lim_{t \to \infty} \frac{\frac{1}{1+\frac{x}{t}}\left(+\frac{x}{\cancel{t^2}}\right)}{+\frac{1}{\cancel{t^2}}}$$

$$= \lim_{t \to \infty} \frac{x}{1+\frac{x}{t}}$$

$$= \frac{x}{1+0}$$

$$\ln L = x$$

$$\cancel{e}^{\cancel{\ln}} L = e^x$$

$$L = e^x \quad \square$$