⑧ Does $\sum_{n=0}^{\infty} \frac{6}{3^{n+2}}$ converge or diverge? If it converges, what is its value?

$$= \sum_{n=0}^{\infty} \left(\frac{6}{3^2}\right)\left(\frac{1}{3^n}\right)$$

$$= \sum_{n=0}^{\infty} \underset{a}{\underset{\uparrow}{\left(\frac{2}{3}\right)}}\underset{r}{\underset{\uparrow}{\left(\frac{1}{3}\right)}}^n$$

Since $\left|\frac{1}{3}\right| < 1$:

$$= \frac{2/3}{1-1/3} = \frac{2/3}{2/3} = \boxed{1} \quad \boxed{\text{converges}}$$

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⑨ Does $\sum_{n=0}^{\infty} 3(-1)^n$ converge or diverge?

Since $|-1| = 1 \geq 1$, the geometric series $\boxed{\text{diverges}}$.

Partial Sum Sequence:

$\langle 3, 3-3, 3-3+3, 3-3+3-3, \ldots \rangle$

$= \langle 3, 0, 3, 0, 3, 0, 3, 0, \ldots \rangle$ diverges

(10) Does $\sum_{i=1}^{\infty} \frac{i+\sin i}{2i}$ converge or diverge?

$$\frac{1}{2} = \lim_{i\to\infty} \frac{i-1}{2i} \leq \lim_{i\to\infty} \frac{i+\sin i}{2i} \leq \lim_{i\to\infty} \frac{i+1}{2i} = \frac{1}{2}$$

Since $\lim_{i\to\infty} \frac{i+\sin i}{2} \neq 0$, $\sum_{i=1}^{\infty} \frac{i+\sin i}{2i}$ $\boxed{\text{diverges}}$

by the Series Divergence Test.

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(11) Suppose $\sum_{n=0}^{\infty} a_n = 3$ and $\sum_{n=0}^{\infty} b_n = 4$. Find $\sum_{n=0}^{\infty} (3a_n - 2b_n)$.

$$= 3\sum_{n=0}^{\infty} a_n - 2\sum_{n=0}^{\infty} b_n = 3(3) - 2(4) = \boxed{1}.$$

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(12) Does $\sum_{k=2}^{\infty} 4\left(\frac{2}{3}\right)^k$ converge or diverge? If it converges, what is its value?

$$= \sum_{k=2-2}^{\infty} 4\left(\frac{2}{3}\right)^{k+2}$$

$$= \sum_{k=0}^{\infty} 4\left(\frac{4}{9}\right)\left(\frac{2}{3}\right)^k$$

$$= \frac{16/9}{1-2/3} = \boxed{\frac{16}{3}}$$ $\boxed{\text{CONV}}$

OR

$$= \left(\sum_{k=0}^{\infty} 4\left(\frac{2}{3}\right)^k\right) - 4\left(\frac{2}{3}\right)^0 - 4\left(\frac{2}{3}\right)^1$$

$$= \frac{4}{1-2/3} - 4 - \frac{8}{3}$$

$$= 12 - \frac{20}{3}$$

$$= \frac{36}{3} - \frac{20}{3} = \boxed{\frac{16}{3}}$$ $\boxed{\text{CONV}}$

(13) Prove $\sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{2}$ using Geo. Series formula proof.

$$S_n = \frac{1}{3} + \cancel{\frac{1}{9}} + \cancel{\frac{1}{27}} + \cancel{\cdots} + \cancel{\frac{1}{3^n}}$$

$$-\left(\frac{1}{3} S_n = \cancel{\frac{1}{9}} + \cancel{\frac{1}{27}} + \cancel{\cdots} + \cancel{\frac{1}{3^n}} + \frac{1}{3^{n+1}}\right)$$

$$S_n - \frac{1}{3} S_n = \frac{1}{3} - \frac{1}{3^{n+1}}$$

$$\frac{2}{3} S_n = \frac{1}{3} - \frac{1}{3^{n+1}}$$

$$S_n = \frac{1}{2} - \frac{3}{2 \cdot 3^{n+1}}$$

$$\sum_{n=1}^{\infty} \frac{1}{3^n} = \lim_{n\to\infty} S_n = \frac{1}{2} - \lim_{n\to\infty} \frac{3}{2 \cdot 3^{n+1}}$$

$$= \frac{1}{2} - 0 = \frac{1}{2} \quad \square$$

(14) Does $\sum_{n=3}^{\infty}\left(\frac{6}{n}-\frac{6}{n+1}\right)$ conv. or div.? If it conv., what's the value?

$$= \lim_{n\to\infty}\left(\frac{6}{3}-\cancel{\frac{6}{4}}\right)+\left(\cancel{\frac{6}{4}}-\cancel{\frac{6}{5}}\right)+\cancel{\cdots}+\left(\cancel{\frac{6}{n}}-\frac{6}{n+1}\right)$$

$$= 2-\lim_{n\to\infty}\frac{6}{n+1} = 2-0 = \boxed{2} \quad \boxed{\text{conv}}$$

(15) Does $\sum_{i=0}^{\infty}\frac{(-3)^i}{2}$ conv. or div.? If it conv., what's the value?

$$= \sum_{i=0}^{\infty}\left(\frac{1}{2}\right)(-3)^i$$

Since $|-3|\geq 1$, the geo. series $\boxed{\text{diverges}}$.

(16) Does $\sum_{n=1}^{\infty}\frac{1}{4^n}$ conv. or div.? If it conv., what's the value?

$$= \sum_{n=0}^{\infty}\frac{1}{4^{n+1}} = \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)^n = \frac{1/4}{1-1/4} = \frac{1/4}{3/4} = \boxed{\frac{1}{3}}$$

$\boxed{\text{conv}}$