① Does $\sum_{k=1}^{\infty} \frac{k^2+4}{(k+2)!}$ converge or diverge?

Ratio Test

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{\frac{(k+1)^2+4}{(k+3)!}}{\frac{k^2+4}{(k+2)!}}\right| = \lim_{k\to\infty} \frac{k^2+2k+5}{\cancel{(k+2)!}(k+3)} \cdot \frac{\cancel{(k+2)!}}{k^2+4}$$

$$= \lim_{k\to\infty} \frac{k^2+2k+5}{k^2+4} \cdot \lim_{k\to\infty} \frac{1}{k+3}$$

$$= 1 \cdot 0$$

$$= 0 < 1$$

$\boxed{\text{Converges}}$

② Does $\sum_{n=0}^{\infty} \frac{(2n)!}{n+3}$ conv or div?

Ratio Test

$$\lim_{n\to\infty}\left|\frac{\frac{(2n+2)!}{n+4}}{\frac{(2n)!}{n+3}}\right| = \lim_{n\to\infty} \frac{\cancel{(2n)!}(2n+1)(2n+2)}{\cancel{n+4}} \cdot \frac{\cancel{n+3}}{\cancel{(2n)!}}$$ diverges

$\boxed{\text{diverges}}$

③ Does $\sum_{m=2}^{\infty} \frac{5^m}{m!}$ converge or diverge?

---

Ratio Test

$$\lim_{m\to\infty} \left| \frac{\frac{5^{m+1}}{(m+1)!}}{\frac{5^m}{m!}} \right| = \lim_{m\to\infty} \frac{\cancel{5^m}\cdot 5^1}{\cancel{m!}(m+1)} \cdot \frac{\cancel{m!}}{\cancel{5^m}} = \lim_{m\to\infty} \frac{5}{m+1} = 0 < 1$$

$\boxed{\text{converges}}$

---

④ Does $\sum_{n=0}^{\infty} (-1)^n \frac{n!}{2^n(n+2)!}$ converge or diverge?

---

$$= \sum_{n=0}^{\infty} (-1)^n \frac{\cancel{n!}}{2^n\cancel{(n!)}(n+1)(n+2)}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1}} \frac{1}{2^{n+1}(n+2)(n+3)}}{\cancel{(-1)^n} \frac{1}{2^n(n+1)(n+2)}} \right| = \lim_{n\to\infty} \frac{\cancel{2^n}(n+1)\cancel{(n+2)}}{\cancel{2^n}(2)\cancel{(n+2)}(n+3)}$$

$$= \lim_{n\to\infty} \frac{(n+1)}{2(n+3)} = \frac{1}{2} < 1$$

$\boxed{\text{Converges}}$

⑤ Does $\sum_{p=0}^{\infty} \frac{3^p}{(p+7)^p}$ converge or diverge?

Root Test

$$\lim_{p\to\infty} \sqrt[p]{\left|\frac{3^p}{(p+7)^p}\right|} = \lim_{p\to\infty} \frac{3}{p+7} = 0 < 1$$

$\boxed{\text{Converges}}$

⑥ Does $\sum_{n=9}^{\infty} \left(1+\frac{2}{n}\right)^{n^2}$ converge or diverge?

Root Test

$$\lim_{n\to\infty} \sqrt[n]{\left|\left(1+\frac{2}{n}\right)^{n^2}\right|} = \lim_{n\to\infty} \left(1+\frac{2}{n}\right)^n = e^2 > 1$$

$\boxed{\text{diverges}}$

(7) Does $\sum_{j=3}^{\infty} (-3)^j \frac{1}{j4^j}$ conv or div?

---

Root Test

$$\lim_{j\to\infty} \sqrt[j]{\left|(+3)^j \frac{1}{j4^j}\right|} = \lim_{j\to\infty} 3^{j/j} \frac{1}{j^{1/j}\, 4^{j/j}} = \frac{3}{4} \lim_{j\to\infty} \frac{1}{j^{1/j}} = \frac{3}{4} \cdot \frac{1}{1} < 1$$

$\boxed{\text{Converges}}$

---

(8) Does $\sum_{n=1}^{\infty} \left(\frac{1-4n^2}{(n+1)(3n+1)}\right)^{n+3}$ conv or div?

---

Root Test

$$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{1-4n^2}{(n+1)(3n+1)}\right)^{n+3}\right|} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{4n^2-1}{(n+1)(3n+1)}\right)^{n+3}}$$

$$= \lim_{n\to\infty} \left(\frac{4n^2-1}{3n^2+4n+1}\right)^{1+3/n}$$

$$= 4/3 > 1$$

$\boxed{\text{diverges}}$

(9) Does $\sum_{n=4}^{\infty} (-1)^{n+1} \frac{ne^{-n}}{(2n+1)\ln(n+1)}$ converge or diverge?

Ratio test

$$\lim_{n\to\infty} \left| \frac{\frac{\cancel{(-1)}(n+1)e^{-n-1}}{(2n+3)\ln(n+2)}}{\frac{\cancel{(-1)}ne^{-n}}{(2n+1)\ln(n+1)}} \right| = \lim_{n\to\infty} \frac{(n+1)}{e^{\cancel{n}+1}(2n+3)\ln(n+2)} \frac{\cancel{e^{n}}(2n+1)\ln(n+1)}{n}$$

$$= \frac{1}{e}\left(\lim_{n\to\infty} \frac{n+1}{n}\right)\left(\lim_{n\to\infty} \frac{2n+1}{2n+3}\right)\left(\lim_{n\to\infty} \frac{\ln(n+1)}{\ln(n+2)}\right)$$

$$= \frac{1}{e}(1)(1)(1) = \frac{1}{e} < 1$$

$\boxed{\text{Converges}}$

(10) Does $\sum_{n=1}^{\infty} \frac{(n-1)!}{10^n}$ converge or diverge?

$\boxed{\text{Ratio Test}}$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+\cancel{1}\cancel{-1})!}{10^{n+1}}}{\frac{(n-1)!}{10^n}} \right| = \lim_{n\to\infty} \frac{n!}{10^{n+1}} \frac{10^n}{(n-1)!}$$

$$= \lim_{n\to\infty} \frac{\cancel{(n-1)!}(n)}{\cancel{10^n}\cdot 10} \frac{\cancel{10^n}}{\cancel{(n-1)!}}$$

$$= \lim_{n\to\infty} \frac{n}{10} \text{ diverges to } \infty > 1$$

$\boxed{\text{diverges}}$

(11) Does $\sum_{k=3}^{\infty} \left(1-\frac{1}{k}\right)^{k^2}$ converge or diverge?

$\boxed{\text{Root Test}}$

$$\lim_{k\to\infty} \sqrt[k]{\left|\left(1-\frac{1}{k}\right)^{k^2}\right|} = \lim_{k\to\infty} \left(1-\frac{1}{k}\right)^{k^2/k} = \lim_{k\to\infty} \left(1+\frac{-1}{k}\right)^k$$

$$= e^{-1} = \frac{1}{e} < 1$$

$\boxed{\text{Converges}}$

⑫ Does $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converge or diverge?

---

Ratio Test

$$\lim_{n\to\infty} \left| \frac{\frac{1}{(n+1)^2}}{\frac{1}{n^2}} \right| = \lim_{n\to\infty} \frac{n^2}{(n+1)^2} = \left( \lim_{n\to\infty} \frac{n}{n+1} \right)^2 = 1^2 = 1$$

inconclusive

(Don't forget easy rules...)

p-Series

$$\sum_{n=2}^{\infty} \frac{1}{n^{2}}$$

$p > 1$

$\boxed{\text{Converges}}$